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ELECTRICAL TESTING.

H. R. KEMPE.

A HANDBOOK

OF

ELECTRICAL TESTING.

BY

H. R. KEMPE,

ASSOCIATE OF THE SOCIETY OF TELEGRAPH ENGINEERS.

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PREFACE.

It was originally intended that the series of papers in the 'Telegraphic Journal,' of which this little book is a revised and enlarged reprint, should have been confined to the subject indicated by their title, viz. "Resistances and their Measurement."

During the progress of the publication, however, it occurred to me that the treatment of the subject might be so extended and amplified as to embrace the general principles of electrical testing.

My aim has been to produce a series of papers which would not only explain the practical application of formulæ given in existing text-books, but also show by what processes they may be obtained.

When an amateur in electrical science, I had experienced considerable difficulty in satisfying myself upon this point, and I know that others have been, and are, similarly in need of information and guidance.

With the view, therefore, of doing something to aid students, and those who may be concerned or interested in the inquiry, the present treatise is offered.

I trust it will meet a want which, so far as I know, has not hitherto been attempted to be supplied.

It is intimated at the commencement of the first chapter that the reader is assumed to possess a certain knowledge of electrical terms and expressions. This assumption rests upon the fact that explanations of these are given in most recent works on electricity (as, e. g. Ferguson's 'Electricity,' Noad's 'Student's Text-Book of Electricity,' Deschanel's 'Electricity and Magnetism'), which do not, however, touch upon the subject of electrical testing; and it is essential that the reader should have made himself acquainted with the general subject before he can proceed to this particular branch of it.

H. R. K.

ST. JAMES'S RECTORY, PICCADILLY,
LONDON, *January*, 1876.

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A HANDBOOK

OF

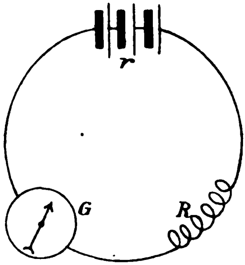
ELECTRICAL TESTING.

CHAPTER I.

SIMPLE TESTING.

THE terms *resistance*, *electromotive force*, *inductive* or *electrostatic capacity*, &c., are now so well understood even by those who have but a slight acquaintance with the subject of Electrical Testing, that it is unnecessary to dwell upon them here; without, therefore, staying to discuss their meaning, we will at once proceed to examine the theory of the simplest method of measuring a resistance.

FIG. 1.



Suppose a battery of a resistance r, and electromotive force E, a galvanometer of a resistance G, and a wire of a resistance R, be "joined up" in circuit, as shown by Fig. 1. By

Ohm's fundamental law, the strength of current C, which will flow out of the battery and through the galvanometer, will be

$$C = \frac{E}{R + r + G}.$$

The current, in flowing through the galvanometer, produces a deflection of its needle, which deflection will remain constant provided the electromotive force of the battery and also the resistances remain constant. If now R be a wire whose resistance we require to find, and which we can replace by another wire the value of whose resistance can be varied at pleasure, then by adjusting this latter so that the deflection of the galvanometer needle becomes the same as it was before the change of resistances was made, this resistance gives the value of our unknown resistance R.

This test, although exceedingly simple, is a very good and accurate one if a little ordinary care be taken in making it. Its correctness is only limited by the sensibility of the galvanometer to small changes of strength in the current affecting it, and by the accuracy with which the variable resistance can be adjusted.

Next, suppose the galvanometer to have its scale so graduated that the number of divisions on it will, by the deflection of the needle, accurately represent the comparative strength (C) of currents which may pass through it. Let the battery, galvanometer, and resistance be joined up as at first, then as before

$$C = \frac{E}{R + r + G}\text{; or, } E = C(R + r + G).$$

Now remove R, and insert any other known resistance ρ, in its place. Calling the new strength of current C_1, then

$$C_1 = \frac{E}{\rho + r + G}\text{; or, } E = C_1(\rho + r + G).$$

Combining these two equations, we get

$$C(R + r + G) = C_1(\rho + r + G),$$

from which

$$R = \frac{C_1}{C}(\rho + r + G) - (r + G). \qquad [1]$$

Now, as we have supposed the deflections of the galvanometer needle to be directly proportional to the strengths of current which produce them, we may, instead of C and C_1, write in our formulæ the deflections of the galvanometer needle which those strengths produce. Calling, then, a° the deflection obtained with the strength C, and a_1° that with the strength C_1, our formula [1] becomes

$$R = \frac{a_1^\circ}{a^\circ}(\rho + r + G) - (r + G). \qquad [2]$$

In order to find R, it is necessary to know G, which is usually marked on the galvanometer by the manufacturer. r also must be known; but as it is difficult to determine its value accurately, it is always best to use a battery whose resistance is very small in comparison with the other resistances in the circuit, and may consequently be neglected, we may therefore write our formula

$$R = \frac{a_1^\circ}{a^\circ} (\rho + G) - G. \qquad [3]$$

Having then obtained a° with R and a_1° with ρ, we can find the value of R.

For example.

With a galvanometer whose resistance was 100 ohms, and a battery whose resistance could be neglected, we obtained with a resistance R a deflection (a°) of 20°, and with a resistance (ρ) of 200 ohms a deflection (a_1°) of 30°. What was the unknown resistance R?

$$R = \frac{30}{20} (200 + 100) - 100 = 350 \text{ ohms.}$$

Next, suppose it is required to find the resistance of a galvanometer. From equation [3] we find that

$$G = \frac{R\,a^\circ - \rho\,a_1^\circ}{a_1^\circ - a^\circ}. \qquad [4]$$

If, then, with a known resistance R we obtain a°, and with a known resistance ρ we obtain a_1°, we can determine G.

For example.

With a galvanometer (G) and a battery whose resistance could be neglected, we obtained with a resistance (R) of 350 ohms a deflection (a°) of 20°, and with a resistance (ρ) of 200 ohms a deflection (a_1°) of 30°. What was the resistance of the galvanometer?

$$G = \frac{350 \times 20 - 200 \times 30}{30 - 20} = 100 \text{ ohms.}$$

Lastly, when the resistance of our battery is considerable, and it is required to find its value, we find from equation [2] that

$$r = \frac{a^\circ R - a_1^\circ \rho}{a_1^\circ - a^\circ} - G. \qquad [5]$$

For example.

With a galvanometer of a resistance (G) of 100 ohms, and a battery (r), we obtained with a resistance (R) in circuit of 300 ohms a deflection (a°) of 30°, and with a resistance (ρ) in circuit of 150 ohms a deflection (a_1°) of 40°. What was the resistance of the battery?

$$r = \frac{30 \times 300 - 40 \times 150}{40 - 30} - 100 = 200 \text{ ohms.}$$

The formulæ may be considerably simplified if we so adjust our resistances that one deflection becomes half the other, or, in other words, if we make $a^\circ = \frac{a_1^\circ}{2}$. Formula [3] for determining any resistance then becomes

$$R = 2\rho + G.$$

Formula [4] for determining the resistance of a galvanometer,

$$G = R - 2\rho.$$

And formula [5] for determining the resistance of a battery,

$$r = R - (2\rho + G).$$

R being in all cases the resistance which gives the small deflection, and ρ the smaller resistance which doubles it.

When the resistance we have to measure is very high compared with the resistance of the galvanometer and battery used for measuring, in our equation

$$R = \frac{a_1^\circ}{a^\circ}(\rho + r + G) - (r + G),$$

we may practically, especially when great accuracy of measurement is not required, put G as well as r equal to 0, in whch case,

$$R = \frac{a_1^\circ}{a^\circ}\rho.$$

To measure a resistance according to this formula, we should first join up, as shown by Fig. 1, our battery, galvanometer, and standard resistance, as it is called, which in our formula is ρ; and having noted the deflection a_1°, should multiply it by ρ; which gives us what is called the *constant.*

R (the resistance to be determined) is then inserted in the place of ρ; a new deflection, a°, is obtained, by which we divide the constant, and thus we obtain R. This method of measuring resistances is the one generally employed in taking the daily tests for *insulation* resistance of telegraph lines, the standard resistance ρ being usually 1000 ohms.

When the insulation resistances of several lines are to be measured, the constant would first be taken and worked out, and the several lines to be measured being inserted one after the other in the place of the resistance ρ, the deflections are noted; and the constant being divided by the several deflections, the resistances are thus obtained.

For example.

With a battery, a galvanometer, and a resistance (ρ) of 1000 ohms in circuit, we obtained a deflection (a_1°) of 20°, then

$$\text{Constant} = 1000 \times 20 = 20000.$$

Taking away our resistance and inserting

Wire No. 1, we obtained a deflection of 5°
„ 2, „ „ 6°
„ 3, „ „ 12°
„ 4, „ „ 3°

The resistances of our wires would then be

No. 1, 20000 ÷ 5° = 4000 ohms
„ 2, 20000 ÷ 6° = 3333 „
„ 3, 20000 ÷ 12° = 1666 „
„ 4, 20000 ÷ 3° = 6666 „

These results are the *total* insulation resistances of the wires, which may be of various lengths. To get comparative results, it is necessary to obtain the insulation resistance of some unit length of each wire, such as a mile.

Now, it will be readily seen that the greater the length of the wire the *greater* will be the leakage, and consequently the *less* will be the insulation resistance, or, in other words, this resistance will vary *inversely* as the length of the wire. To obtain, then, the insulation resistance, or insulation, as it is simply called, all we have to do is to multiply the *total* insulation by the length of the wire. Thus, if No. 1 wire, for example, was 100 miles long, its insulation per mile would be 4000 × 100 = 400,000 ohms. It is usual to fix a standard insulation per mile, and if the result is below that standard, the line is considered faulty. 200,000 ohms per mile is a good standard to take.

This rule of multiplying the total insulation by the mileage of the wire to get the insulation per mile, is not strictly correct, more especially for long lines, as it assumes that the leakage is the same at every point along the line. This, however, is clearly not the case, as a little of the current leaking out at one point leaves a smaller quantity to leak out at the next. In fact, we really measure the last portions of the line with a weaker battery than we do the first. The true law is, however, exceedingly complicated, and we shall not stop to discuss it here.

We should remark that the numbers we have given as degrees are not really intended to be true degrees of deflection, but numbers which represent the strengths of current; for in no galvanometer are the strengths of current proportional to the true degrees of deflection which they produce, but are proportional to some function of those degrees, such as the tangent. Thus, if we were reading off the scale of degrees on a *tangent* galvanometer, that is to say a galvanometer in which the strengths of current are directly proportional to the *tangents* of the angles of deflection which those currents produce, we should have to find the tangents of those degrees of deflection before multiplying and dividing.

For example.

If with a tangent galvanometer we obtained with our standard resistance of 1000 ohms a deflection of 20° (true), and with the unknown resistance (R) a deflection of 15° (true), we should have

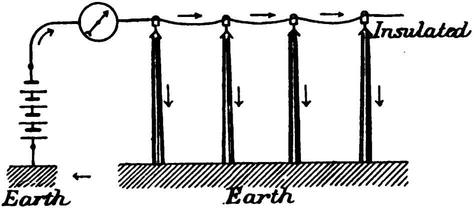
$$R = \frac{\tan 20^\circ \times 1000}{\tan 15^\circ} = \frac{\cdot 364 \times 1000}{\cdot 268} = 1358 \text{ ohms.}$$

In future, when degrees are indicated, we shall mean true degrees.

When measuring the insulation resistance of a line of telegraph, having taken the constant, we should join up our instruments and line, as shown by Fig. 2. In making a measurement of this kind, it is usual to have the positive pole of the battery to earth, so that a negative (zinc) current flows out to the line, as a zinc current will show best any defective insulation in the line, a positive current having the effect, to a certain extent, of *sealing* a fault up, more especially if any underground work which may be in the circuit is defective.

The foregoing method of measurement is, as a rule, sufficiently accurate for all practical purposes. Greater accuracy may, however, be obtained with but little extra trouble by allowing for the resistance of our battery and galvanometer in the following manner:

FIG. 2.



Instead of multiplying the *constant* deflection by the 1000 ohms standard resistance, multiply it by 1000 plus the resistance of the galvanometer and battery, and having divided the result by the deflection obtained with the line wire in circuit, subtract from the result the resistance of the galvanometer and battery.

For example.

With a standard resistance of 1000 ohms, a tangent galvanometer of a resistance of 50 ohms, and a battery of a resistance of 100 ohms, we obtained a deflection of 30°, and with the line wire in circuit a deflection of 10°. What was the insulation resistance of the line?

$$\left.\begin{array}{l}\text{Insulation}\\ \text{resistance}\end{array}\right\} = \frac{\tan 30^\circ\,(1000+50+100)}{\tan 10^\circ} - (50+100)$$

$$= \frac{\cdot 577 \times 1150}{\cdot 176} - 150 = 3760 \text{ ohms.}$$

When a large number of wires have to be measured for insulation daily, it is very convenient to have a table constructed on the following plan:

		α_1°			
		45°	44°	43°	- - - -
α°	1°	58823	56765	54824	- - - -
	2°	28571	27571	26628	- - - -
	3°	19230	18558	17923	- - - -
	⋮	⋮	⋮	⋮	⋮

In this table, the top row of degrees represents the deflections obtained with a tangent galvanometer through a standard resistance of 1000 ohms, the first vertical column the deflections with the line wire in circuit. The numbers at the points of intersection of a vertical with a horizontal column are the resistances corresponding to those deflections, calculated from the formula

$$\left.\begin{array}{l}\text{Insulation}\\ \text{resistance}\end{array}\right\} = \frac{\tan \alpha_1^\circ \times 1000}{\tan \alpha^\circ}.$$

Thus the *constant* deflection with the 1000 ohms standard resistance being 45°, and the deflection with the line wire being 2°, the resistance required we see at a glance to be 28,571 ohms.

Before proceeding to the more intricate systems of measurement, we will consider some of the instruments which would be used in making measurements such as we have described.

CHAPTER II.

THE RESISTANCE COILS.

THE essential points of a good set of resistance coils are, that they should not vary their resistance appreciably through change of temperature, and that they should be accurately adjusted to the standard units, which adjustment ought to be such that not only should each individual coil test according to its marked value, but the total value of all the coils together should be equal to the numerical sum of their marked values. It will be frequently found in imperfectly adjusted coils that although each individual coil may test, as far as can be seen, correctly, yet when tested all together their total value will be one or two units more or less than the sum of their individual values; because, although an error of a fraction of a unit may not be perceptible in testing each coil individually, yet the accumulated error may be comparatively large.

The wire of the coils is, as a rule, of German silver, the specific resistance of which metal is but little affected by variations of temperature. The wire is usually insulated by two coverings of silk, and is wound double on ebonite bobbins, the object of the double winding being to eliminate the extra current which would be induced in the coils if the wire were wound on single. By double winding, the current flows in two opposite directions on the bobbin, the portion in one direction eliminating the inductive effect of the portion in the other direction. When wound, the bobbins are saturated in hot paraffin wax, which thoroughly preserves their insulation, and prevents the silk covering from becoming damp, which would have the effect of partially short-circuiting the coils and thereby reducing their resistance.

The small resistances are made of thick wire, the higher ones of thin wire to economize space.

When bulk and weight are of no consequence, it is better to have all the coils made of thick wire, more especially if high battery power is used in testing, as there is less liability of the coils becoming heated by the passage of the current through them.

The individual resistances of a set of coils are generally of such values that, by properly combining them, any resistance from 1 to 10,000 can be obtained. One arrangement in general use has coils of the following values: 1, 2, 2, 5, 10, 10, 20, 50, 100, 100, 200, 500, 1000, 1000, 2000, 5000 ohms.

These numbers enable any resistance from 1 to 10,000 to be obtained, using a minimum number of coils without fractional values. With these numbers, however, it is a matter of some little difficulty to see at once what coils it is necessary to put into circuit in order to obtain a particular resistance; and as it is often necessary to be quick in changing the resistances, the following numbers are frequently used:—1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400, 1000, 2000, 3000, 4000, which enables any particular resistance, that is required to be inserted, to be seen almost at a glance.

The way in which the different coils are put in circuit is shown by Fig. 3. The ends of the several resistances, *c*, *c*, *c*, · · · · are inserted between the brass blocks, *b*, *b*, *b*, · · · ·. Any of the coils can then be cut out of the circuit between the first and last blocks, by inserting plugs, *p*, as shown, which short-

FIG. 3.

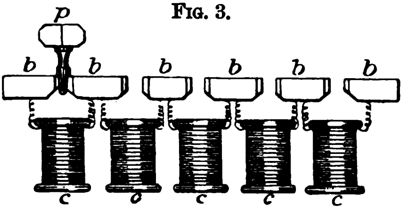
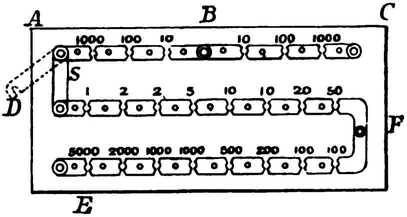


FIG. 4.



circuit the coils between them; thus, if all the plugs were inserted, there would be no resistance in circuit, and if all the plugs are out, all the coils would be in circuit.

There are various ways of arranging the coils in sets; one of the most common is that shown by Fig. 4. This form is much used in submarine cable testing. The brass blocks, here shown in plan, are screwed down to a plate of ebonite which forms the top of the box in which the coils are enclosed. The ebonite bobbins are fixed to the lower surface of the ebonite top, the ends of the wires being fixed to the screws which secure the brass blocks. The holes shown in the middle of the brass blocks are convenient for holding the plugs that are not in use.

It will be seen that six terminals, A, B, C, D, E, F, are provided; when we only require to put a resistance in circuit, the two terminals D and E would be used. The use of the other terminals and of the movable brass strap S, will be explained hereafter.

In using a set of resistance coils, one or two precautions are necessary.

First of all, it should be seen that the brass shanks of the plugs are clean and bright, as the insertion of a dirty plug will not entirely cut out of circuit the coil it is intended to cut out. It is a good plan, before commencing to test, to give the plugs a scrape with a piece of glass or emery paper, taking care to rub off any grains of grit which may remain sticking to them after this has been done.

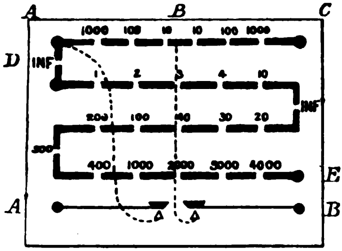
When a plug is inserted, it should not be simply pushed into the hole, but a twisting motion should be given it in doing so, so that good contact may be insured; too much force should not be used, as the ebonite tops may be thereby twisted off in extracting the plugs. Care also should be taken that the neighbouring plugs are not loosened by the fingers catching them during the operation of shifting a plug.

Before commencing work it is as well to give all the plugs a twist in the holes, so as to insure that none of them are left loose. On no account must the plugs be greased to prevent their sticking, and their brass shanks should be touched as little as possible with the fingers.

For taking the insulation resistance of a line in the manner described in the last chapter, such an elaborate set of coils is not of course required. A single coil of a resistance of 1000 ohms in a box with two terminals, to which the ends of the coils are attached, is all that is required.

One of the most useful sets of coils for general purposes is represented in outline by Fig. 5. The general arrangement of resistances, it will be seen, is the same as that shown by Fig. 4. Two keys, however, are provided (drawn in the figure in elevation, for distinctness), the contact point of the left-hand key being connected, as shown by the dotted line, with the middle brass block of the upper set of resistances, the terminal B′ at the end of the key corresponding, in fact, when the key is pressed down, with the terminal B shown in Fig. 4. In like manner the terminal A′ corresponds with the terminal A. In the place of the movable piece of brass between A and D, a plug marked INF. (infinity) is provided, which answers the same purpose; an infinity plug is also placed at the first bend of the coils on the right hand of the figure.

FIG. 5.



When we require simply to insert a resistance in a circuit, we should use the terminals A′ and E, the left-hand key being pressed down when the deflection of the galvanometer needle is to be noted. The current can thus be conveniently cut off or put on when required, by releasing or depressing the key.

Care should be taken that the two *infinity* plugs are firmly in their places, to insure their making good contact. For the same purpose the key contacts should be occasionally touched with emery paper or a fine file.

CHAPTER III.

THE GALVANOMETER.

For the class of tests in which it is required, by adjusting certain resistances, either to bring the needle to zero, or to the same deflection in making two measurements, as described on pages 1 and 2, a galvanometer having its scale graduated to degrees would be sufficient. It should be provided with an astatic pair of needles, suspended by a cocoon fibre, whose end is attached to a screw, by the twisting of which the needles can be lowered down on to the coil, so that when it is not in use there would be no danger of the fibre being fractured by moving the instrument about.

When the instrument is to be used it should be placed on a firm table, and the screw attached to the fibre turned until the needles swing clear of the coil. The instrument should then be placed in such a position that the top needle stands as nearly as possible over the zero points. It should next be carefully levelled by means of the levelling screws attached to its base, until the metal axis which connects the two needles together is exactly in the centre of the hole in the scale-card through which it passes.

The adjustment of the needles to zero is much facilitated in some galvanometers by making the coil movable about the centre of the scale-card by means of a rack and pinion, or even by a simple handle attached direct to the coil. The final touch can thus be given without shaking the needles, which would render exact adjustment difficult.

In the best galvanometers there is a scale graduated to degrees attached to the coil, so that the angle through which it is turned can be seen if required. This scale is employed when using the instrument as a *sine* galvanometer.

We before stated that the strengths of currents producing certain deflections are not directly proportional to those deflections, but to some function of them, such as the *tangent*. In measuring strengths of current by using the instrument as a *sine* galvanometer we proceed as follows:—

The needle is first adjusted to zero. The current whose strength is to be measured is then allowed to flow, and a deflection of the needle produced. The *coil* is now turned round; this causes the needle to diverge still more with respect to the stand of the instrument, but the angle which it makes with the coil becomes less and less the farther it is turned, and finally a point is reached at which the needle is again parallel to the coil—that is, its ends are again over the zero points on the scale-card. The reason of this is, that the deflective action of the coil on the needle is always the same, provided the current strength does not vary, but the farther the needle moves from the magnetic meridian, the greater becomes its tendency to return to that meridian, and finally when the needle becomes parallel to the coil this reactive force of the earth's magnetism exactly balances its deflective force.

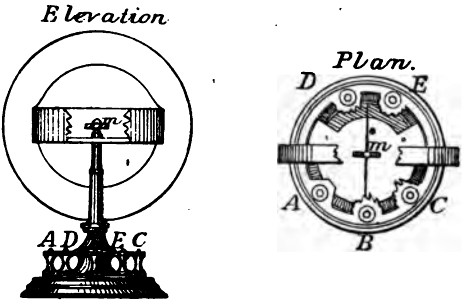
The strength of the current which produces the deflection of the needle will then be directly proportional to the *sine* of the angle through which the coil is turned.

The galvanometer used as a *sine* instrument is extremely accurate if properly managed, its results being entirely independent of the shape of the coil, size of the needle, &c. The only precaution necessary is to see that when the needle is at zero at starting it is brought back exactly to zero. Indeed it is not absolutely necessary that the starting point be zero,—the law of the *sines* holds good if the needle be at, say, 5° when commencing, but in this case the needle must be brought back by the turning of the coil to 5°, and not to zero.

The *tangent* galvanometer, which is perhaps the most useful and convenient form for general purposes, consists essentially of coils of wire wound in a deep groove in the circumference of a circular ring, usually of brass, about 6 or 7 inches in diameter, with a small magnetic needle at its centre moving over a graduated circle. To render the instrument correct, it is necessary that the length of the needle be small compared with the diameter of the coil; about three-quarters of an inch for a 6 or 7 inch ring is a convenient size, and gives sufficiently accurate results for all practical purposes.

Fig. 6 shows a very convenient form of this instrument, used by the Postal Telegraph Department.

FIG. 6.



The magnet *m* has a long pointer, of gilt copper, about 5 inches long, fixed at right angles to it, so that its ends point to the zero of the scales placed at each end of it, when the magnetic needle is parallel to the coil.* The magnet is so placed that its central point is at the axis of the coils and also in the same plane with them.

The object of having the needle small, as compared with the diameter of the coil, is to insure, as far as possible, the magnetic influence of the current on the needle being the same at whatever angle the needle may be with respect to the coil. Theoretically to effect this result, the magnet should be a mere point, but this is of course impossible, and the error is not great when the coil is eight or ten times as large in diameter as the length of the needle. Upon the influence of the coil on the needle being the same, whatever angle the needle takes up with respect to it, depends the truth of the proposition, that the *strengths of currents circulating in the coil are proportional to the tangents of the angles of deviation of the needle.*

---

* Pieces of the instrument are broken away to show the different parts more clearly.

As it would be difficult to read off the angular deflection of the needle from the needle itself, on account of its shortness, the long pointer is attached, which moves over the graduated scales under its extremities.

One of these scales is graduated to true degrees, and the other to numbers proportional to the tangents of those degrees, so that if we read off two deflections from the degrees scale, the other extremity of the pointer will indicate approximately numbers proportional to the tangents of those two degrees of deflection.

Now as the strengths of currents producing certain deflections are proportional to the tangents of the degrees of deflection, if we read off from the degrees scale we must, before working out a formula which has reference to the strengths of currents, reduce the degrees to tangents from a table of tangents. If, however, we read off from the tangent scale, no reduction is necessary, and the numbers can be at once inserted in the formula. When time is no object it is better to use the degrees scale, as it is the more accurate of the two.

To avoid parallax error, in consequence of the needle being elevated above the scale, a piece of looking-glass is provided close to the tangent scale, so that when we look at the end of the needle and see that the reflected image of the pointer coincides with the pointer itself, we know that we are looking at the end of the pointer perpendicularly with the scale.

The instrument being generally provided with a looking-glass only near the tangent scale, it is necessary when reading off from the degrees scale to run the eye along the pointer to the looking-glass end, and see whether the reflected image corresponds with the pointer at that end; if it does, we may be sure that, when we look at the degrees scale, we do so correctly.

It will be seen from the figure that the instrument is provided with five terminals, A, B, C, D, and E.

The two terminals D and E are attached to the ends of a thick wire, making two or three turns only round the ring, and having practically no resistance. To the terminals A and B are attached the ends of a finer wire, about No. 24 or 30 B.W.G., making a larger number of turns, and having altogether a resistance of about 20 ohms. A similar wire is also attached to the terminals B and C, so that if we use the terminals A and B, or B and C, we have 20 ohms in circuit; and, if the terminals A and C, a resistance of 40 ohms in circuit.

The object of having three different coils is to enable us to work with strong or weak currents; thus, with a strong current, we should use the terminals D and E, as there being only a few turns of wire the effect on the needle would not be great. With a weaker current we should use the terminals A and B, or B and C, and with a still weaker one the terminals A and C. The proper terminals to use in making any particular measurement are best found by experiment; the different terminals being tried until a convenient deflection is obtained. If it is found that one set of terminals gives too high and another too low a deflection, the battery power must be varied, or resistance inserted in the circuit. Experience only can determine what it is best to do in different experiments, as no satisfactory rule can be laid down.

Before using the galvanometer it should be seen that the pointer has not become bent, but stands at right angles to the magnet, and that when suspended it turns freely. On no account should the magnet suspension be oiled to make it turn more freely, as quite the opposite effect to what is intended will be produced by so doing. Care should be taken that the scale is in its proper position, so that when the two ends of the pointer are over the zero points, it (the pointer) stands at right angles to the coils.

To obtain the most accurate results in measuring resistances with the tangent galvanometer, it is best so to adjust our resistances or battery-power as to obtain deflections ranging between 30° and 50°, which range is sufficient for most measurements. Deflections above or below these values should be avoided if possible.

The reason for avoiding deflections below 30° is because a small error made in reading off the deflections from the scale will produce a large error when a result is worked out from a formula in which these deflections enter. And, on the other hand, if we take readings above 50°, a large alteration in our resistances will produce but little effect on the galvanometer needle, as the instrument is not as sensitive with high as with low deflections.

That these sources of error exist will perhaps be best shown by examples.

Thus, in the formula given on page 4, viz.:

$$R = \frac{a_1^\circ}{a^\circ}(\rho + G) - G,$$

or, as it would be more correctly written,

$$R = \frac{\tan a_1^\circ}{\tan a^\circ}(\rho + G) - G,$$

taking ρ = 200 ohms and G = like others 100 ohms, supposing we had obtained the deflections $a_1^\circ = 15^\circ$ and $a^\circ = 10^\circ$, we should have

$$R = \frac{\tan 15^\circ}{\tan 10^\circ}(200 + 100) - 100 = 354 \cdot 84 \text{ ohms.}$$

Now suppose, owing to the unsteadiness of the needle, or some other cause, we had read the deflection 15° as 16°, that is, one degree more than it should be, by inserting this value in the formula in the place of 15°, we should get

$$R = 387 \cdot 88 \text{ ohms,}$$

which gives $387 \cdot 88 - 354 \cdot 84 = 33 \cdot 04$, or 11·75 per cent. above the true value of R.

If now, in measuring a resistance, instead of 15° and 10°, we had obtained, by using higher battery-power, deflections of, say, 42° and 31°, we should have

$$R = \frac{\tan 42^\circ}{\tan 31^\circ}(200 + 100) - 100 = 349 \cdot 56 \text{ ohms.}$$

And supposing, as before, that we had read one degree too much, that is, 42° as 43°, then

$$R = 365 \cdot 58,$$

which gives $365 \cdot 58 - 349 \cdot 56 = 16 \cdot 02$, or an error of 4·58 per cent. only, as against 11·75 per cent. in the last case. This, then, shows the advantage of taking readings above 30°.

We have next to show that the error caused by taking readings above 50° is also a large one.

Supposing, with the before-mentioned resistances, we had obtained deflections of 75° and 68°. We should get, by insertion in the formula,

$$R = 352 \cdot 36;$$

and if deflection 75° was read, in error, as 76°, then

$$R = 386 \cdot 15,$$

which gives $386 \cdot 15 - 352 \cdot 36 = 34 \cdot 79$, or 12·43 per cent. above the true value of R, an error as large as the one we had when the readings were taken below 30°.

A Tangent and Sine Table will be found at the end of the book for use with the Tangent and Sine galvanometers.

We will now consider some of the more elaborate measurements which can be made with the tangent, galvanometer, and resistance coils.

CHAPTER IV.

MEASUREMENT OF THE INTERNAL RESISTANCE OF BATTERIES.

First Method.—On page 5 a formula is given for determining the resistance r, of a battery, viz.:

$$r = R - (2\rho + G),$$

where G is the resistance of the galvanometer used, R a resistance which gave a certain deflection on the tangent scale, and ρ a smaller resistance which gave double that deflection. The formula may perhaps be rendered clearer by expressing it in words, thus: Join up a resistance box (as the set of resistance coils is frequently termed) a tangent galvanometer, and the battery whose resistance is required in simple circuit. Adjust the resistance coils until we get, say 35 divisions (about 32°) deflection on the *tangent* scale of the galvanometer. Note the resistance (R) in the resistance box. Now decrease the resistance until the deflection on the tangent scale becomes 70 (about 50°), that is to say, twice 35. Again note the resistance (ρ) in the box. Then, to get the resistance of the battery: *Double the smaller resistance; to the result add the resistance* *of the galvanometer, and deduct this total from the greater resistance.*

For example.

With a tangent galvanometer whose resistance (G) was 100 ohms, and a battery whose resistance (r) was to be determined, we obtained with a resistance (ρ) in the resistance box of 150 ohms a deflection of 40 divisions on the tangent scale, and on increasing ρ to 600 ohms (R), we obtained a deflection of 20 divisions. What was the resistance of the battery?

$$r = 600 - (2 \times 150 + 100) = 200 \text{ ohms.}$$

Second Method.—Join up the resistance box, galvanometer, and battery, as shown by Figs. 7 and 8. In these figures the upper one shows the theoretical method of joining up, and the lower one the practical method. If we were working with the set of coils shown by Fig. 4, Chapter II., the terminal of the galvanometer joined to B′ would be joined instead to C. We could, of course, adopt the same arrangement in the set of coils shown by Fig. 5, but then we should lose the advantage of having the key in circuit which enables us to cut off the current when we please.

The theory of this method of measurement is as follows: The galvanometer, a resistance box (Res.), and a battery whose resistance (r) is required, are joined up in simple circuit with a *Shunt* (as it is called) S, between the poles of the battery; a deflection of the galvanometer needle is produced with a resistance ρ in the resistance box. The Shunt is now removed by taking out the left-hand infinity

Fig. 7.

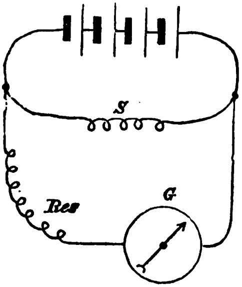
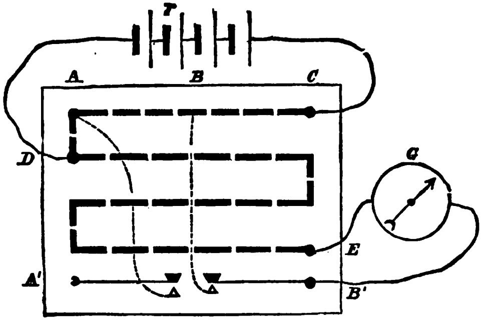


Fig. 8.



plug. This causes the deflection to become larger. ρ is then increased until the deflection becomes the same as it was at first. Let the new Res. be R.

To obtain our formula, we must first consider the law of the joint resistance of two or more parallel circuits.

If we have several wires whose resistances are R_1, R_2, R_3 . . . . respectively, then, conductivity being the inverse or reciprocal of resistance, their conductivities may be represented by $\frac{1}{R_1}, \frac{1}{R_2}, \frac{1}{R_3} \dots$ Now the joint conductivity of any number of wires is simply the sum of their respective conductivities. Thus, *two* wires of equal conductivities evidently will conduct *twice* as well as one of them, when joined parallel to one another; and in like manner, *three* wires will conduct *three* times as well as one. Similarly, two wires, one of which has a conductivity of 2, will, when combined with one which has a conductivity of 1, produce a conductivity of 2 + 1 or 3, for this is simply the same as joining up three wires, each having a conductivity of 1; and so with any number of wires.

Therefore the joint *conductivity* of the several resistances, or multiple arc as it is called, will be $\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$ and conductivity being, as we have said, the reciprocal of resistance, the resistance of the wires will be the reciprocal of this sum, or

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

A particular case of these combinations is that of the joint resistance of two resistances, thus

$$\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}=\frac{R_1 R_2}{R_1+R_2}.$$

or, *the joint resistance of two resistances joined parallel to one another is equal to their product, divided by their sum.*

Applying this law, and taking E to be the electromotive force of the battery and C the strength of the current passing out of it, then

$$C=\frac{E}{r+\frac{S(\rho+G)}{S+(\rho+G)}}.$$

This current divides into two parts, one part going through S, and the other part through the resistance box and the galvanometer. It does this in the inverse proportion of the resistance of those circuits, the part going through the galvanometer being

$$C'=\frac{E}{r+\frac{S(\rho+G)}{S+(\rho+G)}}\times\frac{S}{S+(\rho+G)}.$$

$$=\frac{E\,S}{r(S+\rho+G)+S(\rho+G)}$$

The shunt S is now removed; this causes the deflection of the galvanometer needle to be increased. ρ is now increased to R until the deflection becomes the same as it was previous to the removal of the shunt, or, in other words, until the strength of the current passing through the galvanometer is C′; then

$$C' = \frac{E}{r + R + G},$$

whence

$$\frac{E}{r + R + G} = \frac{E\,S}{r\,(S + \rho + G) + S\,(\rho + G)}.$$

By multiplying up and cancelling,

$$r\,(\rho + G) = S\,(R - \rho),$$

or

$$r = S\,\frac{R - \rho}{\rho + G}.$$

This method was devised by Sir William Thomson.

For example.

With a galvanometer whose resistance (G) was 100 ohms, and a battery whose resistance (r) was to be determined, we obtained with a resistance (ρ) in the resistance box of 500 ohms and a shunt (S) whose resistance was 1000 ohms, a deflection of 30°. On removing the shunt it was necessary, in order to again bring the needle to 30°, to increase ρ to 620 ohms (R). What was the resistance of the battery?

$$r = 1000\,\frac{620 - 500}{500 + 100} = 200 \text{ ohms.}$$

If we adjust ρ, in the first place, so that together with G it equals S, we get the simplification

$$r = S\,\frac{R - \rho}{S} = R - \rho,$$

that is to say, the resistance of the battery is equal to the resistance which was added to ρ in order to make the deflection of the needle the same as it was previous to the removal of the shunt.

For example.

With a galvanometer whose resistance (G) was 100 ohms, and a battery whose resistance (r) was to be determined, we obtained a deflection of 25°, using a resistance ρ of 900 ohms, which, together with the resistance of the galvanometer, made a total resistance equal in value to that of the shunt, viz. 1000 ohms. On removing the shunt, it was found necessary, in order to again bring the needle to 25°, to increase ρ to 1100 ohms (R), that is, to add 200 ohms to it, which was therefore the resistance of the battery.

In making this test practically, we should proceed thus:—Join up the instruments, as shown by Fig. 8, taking care that the two infinity plugs are firmly in their places. Plug up the three holes between B and C, and remove the necessary plugs between A and B; as there are only three plugs here, our choice of a resistance is limited to 1000, 100, or 10, but these are, as a rule, sufficient for all purposes. Now depress the right-hand key, and proceed to remove plugs from between D and E until a convenient deflection of the galvanometer needle is obtained. The galvanometer should be gently tapped with the finger in order to see that the needle is properly deflected and is not sticking, as it is very liable to do, especially when a compass suspended needle is used.

The oscillations of the needle may be arrested by a skilful manipulation of the key; slightly raising it when the needle swings under the influence of the current and again depressing it when it recoils.

The needle being steadily deflected, and the resistance in the box noted, the left-hand infinity plug must be removed, and the resistance between D and E increased until the deflection becomes the same as it was at first, and the resistance being noted, the formula is worked out. When adopting the simplified method, the resistance between D and E would be adjusted in the first place previous to the depression of the key.

To obtain accurate results, it is advisable not to have the resistance between A and B disproportionate to the battery resistance. Thus, if we know that our battery had a resistance somewhere about 50 ohms (the measurement being taken, of course, to obtain its exact value), it would be preferable to use the 100 ohms shunt rather than the one of 1000 ohms, or 10 ohms. Also we should use a resistance between D and E of, say, 700 ohms, in preference to one of 7000 ohms; of course this latter adjustment will be considerably regulated by the power of the battery and the sensitiveness of the galvanometer.

If we have no idea of what value the resistance of the battery is likely to be, the best plan is to use, say, a 100 ohms shunt, and to make a measurement with that. Then if the result shows that the resistance of the battery is, say, 900 ohms, substitute the 1000 ohms in the place of the 100 ohms, and make a fresh measurement which will give the resistance of the battery more accurately.

The internal resistance of batteries can be determined in one or two other ways besides those we have given, but as these methods involve principles which we have not yet discussed, we shall defer their consideration to a later period.

CHAPTER V.

COMPARISON OF THE ELECTROMOTIVE FORCES OF BATTERIES.

THE methods of comparing the electromotive forces of batteries are perhaps more numerous than any other class of measurements. We shall consider the principal of them.

As no absolute standard of electromotive force exists, we cannot directly determine the electromotive force of any particular battery in terms of the standard unit (the *volt*), but can only compare the relative electromotive forces of two or more batteries.

First Method.—Suppose we have two batteries, X and Y, whose electromotive forces are to be compared. Join up battery X with a galvanometer and resistance box in simple circuit, as shown by Fig. 8 (p. 32). All the plugs between A and C being inserted, the infinity plug between A and D being removed, and the connections being made, depress the right-hand key, and remove a sufficient number of plugs from between D and E to obtain a convenient deflection on the tangent scale of the galvanometer. Note this deflection—let it be d divisions; and also note the *total* resistance (R) in circuit—that is, the resistance between D and E, plus the resistance of the galvanometer, plus the resistance of the battery (which must be determined beforehand). Now remove battery X and insert battery Y in its place, and if this battery has a different resistance to X, readjust between D and E so that total resistance in circuit is the same as it was at first. Again note the deflection of the galvanometer needle—let it be d_1 divisions. Then electromotive force of X will be to electromotive force of Y as d is to d_1. For calling E the electromotive force of X, and E_1 the electromotive force of Y, then

$$C = \frac{E}{R} \quad \text{and} \quad C_1 = \frac{E_1}{R},$$

therefore

$$E : E_1 :: C : C_1,$$

and d and d_1 are directly proportional to C and C_1.

For example.

With a tangent galvanometer, whose resistance was 100 ohms, and battery X, whose resistance was 70 ohms, we obtained, with a resistance of 1830 ohms (total, $100 + 70 + 1830 = 2000$), in the resistance box, a deflection of 50 divisions on the tangent scale of the galvanometer; and with battery Y, whose resistance was 50 ohms, we obtained, with a resistance of 1850 ohms (total, $100 + 50 + 1850 = 2000$, as before), in the resistance box, a deflection of 40 divisions; then

$$\text{*E.M.F. of X} : \text{E.M.F. of Y} :: 50 : 40, \\ \text{or as } 5 : 4.$$

* (E. M. F. = electromotive force.)

If the resistances of the batteries whose electromotive forces are to be compared are very small, we may, by using a very high resistance, practically regard the total resistance in circuit as the same, whatever battery we use. The deflections then obtained with any number of different batteries will represent their comparative electromotive forces. The galvanometer will, of course, have to be a comparatively sensitive one.

Second Method.—Join up as in last method, and having noted the deflection and total resistance in circuit (R) with battery X, remove it and insert battery Y in its place. Now readjust resistance between D and E, until the deflection of the galvanometer needle becomes the same as it was at first. Note the resistance in circuit (R); then

$$C = \frac{E}{R} \text{ and } C = \frac{E_1}{R_1},$$

that is,

$$E : E_1 :: R : R_1,$$

or the electromotive forces of the batteries are directly as the total resistances that are in circuit with the respective batteries.

For example.

With a galvanometer whose resistance was 100 ohms, and battery X whose resistance was 50 ohms, we obtained, with a resistance of 2350 ohms (total, 100 + 50 + 2350 = 2500), in the resistance box, a deflection of 40°; and with battery Y, whose resistance was 70 ohms, it was necessary, in order to bring the galvanometer needle again to 40°, to have a resistance of 1830 ohms (total, $100 + 70 + 1830 = 2000$), in the resistance box; then

$$\text{E.M.F. of X} : \text{E.M.F. of Y} :: 2500 : 2000,$$
$$\text{or as } 5 : 4.$$

Third Method.—In Fig. 8 join the zinc pole of battery X to D, as shown, and the other pole to the zinc pole of battery Y, whose other pole in turn is to be joined to C. Adjust resistance so as to obtain a high deflection on the tangent scale of the galvanometer—let it be d divisions; then

$$C = \frac{E + E_1}{R},$$

where R is the total resistance in the circuit. Now reverse battery Y (the weaker one) so that the two oppose one another,—we shall then get a smaller deflection of d_1 divisions; then

$$C_1 = \frac{E - E_1}{R}.$$

From these two equations we get

$$E\,C - E_1\,C = E\,C_1 + E_1\,C_1,$$

that is,

$$E : E_1 :: C + C_1 : C - C_1,$$

or, substituting deflections for strengths,

$$E : E_1 :: d + d_1 : d - d_1.$$

For example.

Two batteries X and Y being joined up together in simple circuit, we obtained, by adjusting resistance in resistance box, a deflection (d) of 72 divisions on the tangent scale of the galvanometer; and with the same resistance in circuit we obtained, on reversing battery Y, a deflection (d_1) of 8 divisions; then

$$\text{E.M.F. of X} : \text{E.M.F. of Y} :: 72 + 8 : 72 - 8,$$
$$\text{or as } 80 : 64,$$
$$\text{that is, as } 5 : 4.$$

Fourth Method.—The most elegant method of comparing the electromotive forces of batteries is that of the late Sir Charles Wheatstone. Battery X, whose resistance we will call r, is joined up in simple circuit with a galvanometer, whose resistance is G, and a resistance R in the resistance box. A deflection of a° is obtained; then (substituting deflections for strengths)

$$a^\circ = \frac{E}{R + r + G}. \qquad [1]$$

R is now increased by ρ, and a new deflection, β°, is obtained; then

$$\beta^\circ = \frac{E}{R + \rho + r + G}. \qquad [2]$$

Battery X is now removed, and battery Y, whose resistance is r_1, substituted in its place. The resistance box is then adjusted to R_1, so that deflection becomes a° as at first; then

$$a^\circ = \frac{E_1}{R_1 + r_1 + G}. \qquad [3]$$

R_1 is now increased by ρ_1, until deflection becomes β°; then

$$\beta^\circ = \frac{E_1}{R_1 + \rho_1 + r_1 + G}. \qquad [4]$$

From equations [1] and [3] we get

$$\frac{E}{(R + r + G)} = \frac{E_1}{R_1 + r_1 + G},$$

or

$$\frac{R + r + G}{E} = \frac{R_1 + r_1 + G}{E_1}; \qquad [5]$$

and from [2] and [4]

$$\frac{E}{\rho + R + r + G} = \frac{E_1}{\rho_1 + R_1 + r_1 + G},$$

or

$$\frac{\rho}{E} + \frac{R + r + G}{E} = \frac{\rho_1}{E_1} + \frac{R_1 + r_1 + G}{E_1}.$$

Substituting the value of

$$\frac{R + r + G}{E},$$

from equation [5] we get

$$\frac{\rho}{E} = \frac{\rho_1}{E_1}.$$

or

$$E : E_1 :: \rho : \rho_1.$$

That is to say, the electromotive forces of the batteries are directly proportional to the added resistances which, in both cases, were required to bring the deflections of the galvanometer needle from a° down to β°.

For example.

With a galvanometer and battery X we obtained, with a resistance of 1830 ohms in the resistance box, a deflection of 40°, and by adding 300 ohms, a deflection of 34°. Battery Y being inserted in the place of X, a resistance of 2350 ohms was inserted in the resistance box, which brought the galvanometer needle to 40° as at first, and by adding 240 ohms, the deflection was reduced to 34° as at first; then

$$\text{E.M.F. of X} : \text{E.M.F. of Y} :: 300 : 240, \quad \text{or as } 5 : 4.$$

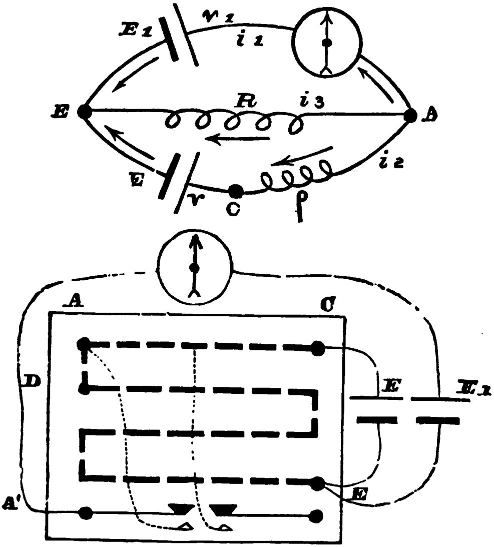
In all these methods of comparing electromotive forces, action goes on in the batteries whilst the measurements are being made.

Now certain batteries fall off considerably in power when continuously worked. If, therefore, it is required to compare the electromotive forces of such batteries when they are not in action, we must adopt some method in which, by a peculiar arrangement of resistances, one battery is made to balance the other. One of these methods is that of Poggendorff. Fig. 9 shows the theoretical and the practical way of making this test.

The theory of this method is best explained by two of Kirchoff's laws, which, though exceedingly simple and easy of application, are not so well known as they ought to be. We will therefore briefly state them:

1. *The sum of the current strengths in all those wires which meet in a point is equal to nothing.*
2. *The sum of all the products of the current strengths and resistances in all the wires forming an enclosed figure equals the sum of all the electromotive forces in the same circuit.*

Fig. 9.



In the figure, E and E_1 are the electromotive forces to be compared. R and ρ are adjustable resistances, r and r_1 being the resistances of the batteries. G is the resistance of the galvanometer. Before equilibrium is obtained, we have

$$(r_1 + G)\,i_1 + R\,i_3 - E_1 = 0 \qquad [1]$$
$$(r + \rho)\,i_2 + R\,i_3 - E = 0 \qquad [2]$$
$$i_1 + i_2 - i_3 0 = 0$$

ρ being any resistance, R is adjusted until the needle of the galvanometer is brought to zero, that is, until

$$i_1 = 0, \text{ which makes } i_2 = i_3.$$

These values being substituted in equations [1] and [2], we get

$$R\,i_3 - E_1 = 0 \text{ or } i_3 = \frac{E_1}{R},$$

and

$$i_3(r + \rho + R) - E = 0;$$

therefore

$$\frac{E_1}{R}(r + \rho + R) - E = 0,$$

or

$$E : E_1 :: (r + \rho) + R : R.$$

It will be observed that in order to get the ratio of E to E_1 from this proportion, we must know the resistance r of the lower battery. If, however, we change ρ to ρ_1, and again get equilibrium by readjusting R to R_1, we get a second proportion, viz.

$$E : E_1 :: (r + \rho_1) + R_1 : R_1.$$

Combining these two proportions, r is eliminated, and we get

$$E : E_1 :: (R - R_1) + (\rho - \rho_1) : (R - R_1),$$

a proportion in which differences of resistance alone enter. In fact $(R - R_1)$ and $(\rho - \rho_1)$ are merely the resistances which we *subtracted* (for R is greater than R_1, and ρ than ρ_1) from the two branches in order to get equilibrium a second time. We have, therefore, simply first to get equilibrium with any resistances we please, then to subtract a resistance from one of the branches, and again get equilibrium by subtracting a resistance from the other. The electromotive forces of the batteries will then be in the proportion of the larger subtracted resistance to the sum of the two subtracted resistances.

For example.

Two batteries, X and Y, whose electromotive forces, E and E_1, were to be compared, were joined up in circuit with a galvanometer and two resistances, as shown by Fig. 9. The resistance ρ being 200 ohms, it was necessary, in order to produce equilibrium, to make $R = 1080$ ohms. ρ was then reduced by 100 ohms, and in order to again produce equilibrium, R had to be reduced by 400 ohms; then

$$\text{E.M.F. of X} : \text{E.M.F. of Y} :: 400 + 100 : 400,$$
$$\text{or as } 5 : 4.$$

In making this test practically, having made the connections as shown by the figure, we should depress the left-hand key, and then, according to the example, we should take out the two 100 plugs between A and C, and proceed to adjust between D and E. This being done, we should replace one of the 100 plugs between A and C, and then proceed to adjust again between D and E, carefully taking note of what resistance we subtract to again produce equilibrium. The electromotive forces of the batteries will then be as shown in the example.

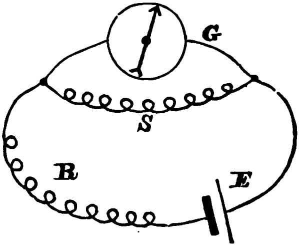
As only one of the batteries (the smaller) in this test has its electromotive force balanced, the other one should be a constant battery, whose electromotive force does not fall off on being worked continuously, such as a Daniell.

CHAPTER VI.

GALVANOMETER SHUNTS.

In making certain measurements we sometimes find that, owing to the sensitiveness of the galvanometer, we are unable to obtain a readable deflection, from the needle being deflected up to the stops. We may reduce this sensitiveness by the insertion of a Shunt between the terminals of the instrument. This arrangement is shown by Fig. 10.

Fig. 10.



If it is required to reduce the strength of current which ordinarily passes through the galvanometer to any proportional part of that current, we must calculate, from the resistance of the galvanometer, what the resistance of the shunt should be to effect that purpose.

Now if we call C the current passing through the galvanometer without a shunt, as we have before explained, on introducing the shunt C will divide between the two resistances, and the greater portion of the current will go through the smaller resistance, and the smaller portion through the greater. Thus if we divide the total current, which passes from one terminal of the galvanometer to the other, into $G + S$ parts, $\frac{G}{G+S}$ parts will go through the shunt, and $\frac{S}{G+S}$ parts through the galvanometer; that is to say, the current going through shunt will be

$$C\frac{G}{G+S},$$

and current going through galvanometer,

$$C\frac{S}{G+S}.$$

If, in this last quantity, we put $S = G$, then current going through galvanometer will be

$$C\frac{G}{G+G} = \frac{C}{2}.$$

Again, if we make $S = \frac{G}{2}$, current going through galvanometer will be

Once more, if S be made equal to $\frac{G}{3}$, current going through galvanometer will be

$$C\frac{\frac{G}{3}}{G+\frac{G}{3}}=\frac{C}{4}.$$

Finally, if S be made equal to $\frac{G}{n-1}$, current going through galvanometer will be

$$C\frac{\frac{G}{n-1}}{G+\frac{G}{n-1}}=\frac{C}{n}.$$

From this it is evident that, to reduce the current flowing through the galvanometer to its $\frac{1}{n}$th part, we must insert a shunt whose resistance is $\frac{1}{n-1}$th part of the resistance of the galvanometer.

In many galvanometers three shunts are provided, which enable us to reduce the strength of current flowing through it to its $\frac{1}{10}$th, $\frac{1}{100}$th, or $\frac{1}{1000}$th part. From what has been said it will be evident that the resistances of the shunts necessary to produce these results will have to be respectively the $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th part of the resistance of the galvanometer.

We are thus enabled to reduce the sensitiveness of a galvanometer to any proportion we wish.

Suppose now, in making an experiment, we placed a resistance box for a shunt between the terminals of the galvanometer, and then adjusted it until we got a convenient deflection for the purpose we required; what deflection should we get on removing the shunt? Let us call C, as before, the current which passes through the galvanometer when no shunt is inserted, and let C_1 be the current which flows through it when the shunt is inserted, then the current which flows through the shunt will be

$$C - C_1.$$

Now the two currents will flow through the shunt and galvanometer in the inverse proportion of their resistances, that is,

$$C_1 : C - C_1 :: S : G,$$

therefore,

$$C = C_1 \times \frac{G + S}{S}.$$

Or expressed in words, we should say that the current which would flow through the galvanometer, when the shunt was removed, would be $\frac{\textit{Galvanometer} + \textit{Shunt}}{\textit{Shunt}}$ times the strength of the current which flows when the shunt is inserted. This proportion is often called the *multiplying power* of a shunt.

It will be noticed in a circuit like that shown by Fig. 10, that when a shunt is introduced between the terminals of the galvanometer which reduces its sensitiveness to $\frac{1}{2}$, or a shunt having a resistance equal to that of the galvanometer, it will not exactly halve the current passing through the galvanometer. If we used a tangent galvanometer, we should find, if the deflection without the shunt was 40 divisions on the tangent scale, the introduction of the shunt would not bring the deflection down to 20, but to some deflection greater than 20. The reason of this is, that the introduction of the shunt reduces the total resistance in the battery circuit, and consequently increases the strength of the current passing out of the battery. It is this increased current, then, which splits between the galvanometer and shunt, and not the original current. To make up for this decreased resistance caused by the introduction of the shunt, it is necessary to add in the battery circuit a *compensating resistance* equal in value to the amount by which the original resistance was reduced. This is thus obtained:—

The resistance between the terminals of the galvanometer before the introduction of the shunt being G, the introduction of the shunt reduces the resistance to $\frac{G\,S}{G+S}$. The resistance, therefore, to be added in the battery circuit will be

$$G - \frac{G\,S}{G+S} = \frac{G^2}{G+S}.$$

Now as S is usually made some fractional value of the galvanometer resistance, say the $\frac{1}{n-1}$th part (which value would be used in reducing the sensitiveness of the galvanometer to $\frac{1}{n}$th), we may write the value of the compensating resistance as

$$\frac{G^2}{G + \frac{G}{n-1}} = G\,\frac{n-1}{n}.$$

For example.

With a galvanometer (G), whose resistance was 100 ohms, it was required to reduce its sensitiveness to $\frac{1}{5}$th. What should be the resistance of the shunt and of the compensating resistance?

Resistance of shunt equals

$$100 \times \frac{1}{5-1} = 25,$$

and compensating resistance equals

$$100 \times \frac{5-1}{5} = 80.$$

A galvanometer of a resistance G, with a shunt inserted and compensating resistance added, may be regarded as a galvanometer whose resistance is also G, but whose sensitiveness is diminished. It would be as well if all galvanometers provided with shunts were also provided with compensating resistances, as calculation would be considerably simplified thereby in a large number of measurements.

We have before stated that the deviations in degrees of the needle of a galvanometer are not directly proportional to the currents that produce them. We may, however, by the help of the points we have just considered, so graduate or *calibrate* the scale of our galvanometer, no matter what kind it is, that the deflections of the needle will at once tell us what the relative strengths of the currents producing those deflections are.

First calculate from the known resistance of the galvanometer, the resistance of shunts required to reduce the amount of current passing through the galvanometer when no shunt is inserted, to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., the amount passing when a shunt is inserted; the resistance of the galvanometer being called 1. Then the resistance of the shunts necessary to reduce the current to

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{n}\text{th}, \&c.$$

will, as we have shown, be

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{n-1}\text{th}$$

the resistance of the galvanometer. Now, as we have also shown, the insertion of shunts reduces the resistance of the circuit in which the galvanometer is placed; we must therefore also calculate the resistances necessary to be inserted in the circuit in order to compensate for the reduction of resistance which takes place when a shunt is inserted. This we do from the formula,

$$x = \frac{G^2}{G+S},$$

where x is the required resistance with the shunt S, G being the galvanometer resistance.

The shunts and their compensating resistances being calculated, to calibrate the galvanometer we proceed as follows:

Join up the galvanometer, a resistance box, and a battery in circuit. First insert the $\frac{1}{2}$ shunt, that is, the shunt equal in resistance to the galvanometer, and also the corresponding compensating resistance in the resistance box. Sufficient resistance is now added in the resistance box to bring the deflection down to, say, 1°; the shunt and compensating resistance are now removed, and as the resistance in circuit is the same as before, and also the whole of the current passing in the circuit now passes through the galvanometer, the strength of current affecting it is exactly double that which deflected the needle originally; the deflection of the needle, therefore, now represents a strength of current double that of the previous experiment. We now insert the $\frac{1}{3}$ shunt and its compensating resistance, and by again adjusting the resistance coils, obtain a deflection of 1°; on now removing the shunt and compensating resistance we get three times the strength of current passing through the galvanometer: the deflection now obtained will therefore represent that strength, and so by inserting

$$\tfrac{1}{4}, \tfrac{1}{5}, \dots \tfrac{1}{n}\text{th}$$

shunts one after another, and repeating the process described, we can get the deflections corresponding to strengths of current equal to 1, 2, 3, 4,...n, and the scale can be marked correspondingly; or these deflections and the corresponding currents producing them can be embodied in a table, so that by referring to it we can at once see the relative power of various currents giving different deviations of the needle.

It will probably be found that up to between 20° and 30° the degrees of deflection are directly proportional to the strengths of the currents producing them; beyond 30°, however, they can no longer be regarded as directly representing the strengths of the currents without considerable error.

It is evident that if, in making a measurement, we want to reduce the deflection of the galvanometer to a readable deflection, we can do so, either by placing a large resistance in the battery circuit, or by introducing a shunt between the terminals of the galvanometer. It is possible, of course, also to produce the same effect by placing a shunt between the poles of the battery, but this method causes a need-less waste of the material of the battery, and also interferes with its constancy.

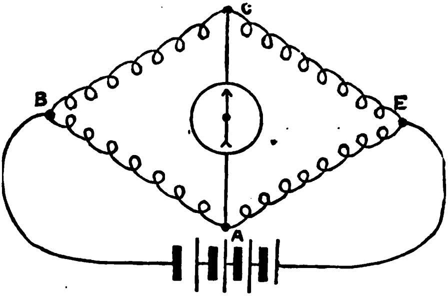
It may be stated as a general rule, but especially when measuring *small* resistances, that it is better to reduce the sensitiveness of a galvanometer by the use of a shunt, rather than by introducing resistances in the battery circuit.

CHAPTER VII.

THE WHEATSTONE BRIDGE.

THE theoretical arrangement of the Wheatstone Bridge, or Balance, is shown by Fig. 11. It consists of four resistances A B, B C, C E, and E A, arranged in the form of a parallelogram, a battery occupying the place of one, and a galvanometer the place of the other, diameter. When the four resistances are so adjusted that equilibrium is produced, that is to say, when no current passes through the galvanometer, then these resistances bear a certain relation to one another. This relation may be thus determined:—

FIG. 11.



When equilibrium is produced, the points A and

C have the same potential; the galvanometer may, therefore, be removed without altering the strengths of current in the other parts of the bridge; and, further, we may join the points A and C without affecting the strengths. Let us first suppose the points A and C to be separated; then the joint resistance given by the four resistances between the points B and E will be

$$\frac{(BC + CE)(AB + EA)}{BC + CE + AB + EA}.$$

If, now, we join A and C, the resistance may be written

$$\frac{BC \,.\, AB}{BC + AB} + \frac{CE \,.\, EA}{CE + EA},$$

which is equal to the former expression, that is,

$$\frac{(BC + CE)(AB + EA)}{BC + CE + AB + EA} = \frac{BC \,.\, AB}{BC + AB} + \frac{CE \,.\, EA}{CE + EA}.$$

By multiplying up and simplifying, we get

$$2 BC \,.\, AB \,.\, CE \,.\, EA = (BC \,.\, AB)^2 + (CE \,.\, EA)^2,$$

from which

$$\frac{BC}{AB} = \frac{CE}{EA}.$$

If, now, three of the quantities in this equation are known, the fourth can be determined; thus:

$$EA = CE \frac{AB}{BC}.$$

In the most general form of bridge, two of the resistances are fixed, and a third is adjustable, the fourth being the resistance whose value is to be determined.

As a rule, we should make A B and B C the fixed resistances, C E the resistance whose value it is required to find, and E A the adjustable resistance.

In the simplest method of measuring we should make A B and B C of equal value, in which case

$$E A = C E;$$

that is to say, the resistance which is between A and E when equilibrium is produced gives the value of C E, the unknown resistance.

It is absolutely necessary that there be some resistance in B C and A B, for otherwise the galvanometer is short-circuited, and equilibrium, as far as the galvanometer is concerned, will be always produced, no matter what resistances we have in the other two branches.

It may perhaps be asked, What are the best resistances to employ for B C and A B? This depends upon the resistance of the galvanometer, and upon the resistance being measured. Of course in the majority of cases we should be unable to make a choice, as we do not know what the value of the resistance to be measured is likely to turn out. In some cases, however, the object in view is to determine *exactly* the value of some resistance whose *approximate* value is known. We may in this case employ certain definite resistances in A B and B C with advantage, as we may thereby make the galvanometer as sensitive as possible.

As a general rule, it may be stated that the joint value of the resistances should not differ much from the resistance of the galvanometer. The rule for a maximum of sensitiveness, when equilibrium is very nearly produced, is that the resistance of the galvanometer should be equal to the joint resistance of the resistances on either side of it, or that

$$G = \frac{(AB + BC)(CE + EA)}{AB + BC + CE + EA};$$

or, when A B and B C are equal (in which case C E and E A will also be equal), then

$$G = 2\frac{AB \,.\, EA}{AB + EA}.$$

As we cannot adjust our galvanometer resistance, our aim would be to find what A B should be. From the above,

$$CB \text{ and } AB = \frac{G \,.\, EA}{2\,EA - G}.$$

For example.

In measuring the exact value of a resistance which we knew to be about 300, we employed a galvanometer whose resistance was 150 ohms. What would be the best resistances to employ in the branches A B and C B, supposing that they were equal?

$$\text{C B and A B} = \frac{150 \times 300}{2 \times 300 - 150} = 100.$$

Besides using equal resistances in A B and B C, we can make one of the two to be 10 or 100 times as great as the other, or, in fact, any multiple of it we like, but multiples of 10 are those most commonly used. If, when we are measuring a resistance C E, we make A B 10 times as large as B C, then every unit of resistance in E A represents $\frac{1}{10}$th of a unit in C E, for in this case

$$\text{C E} = \frac{\text{E A}}{10}.$$

We can, therefore, by this device determine the value of a resistance to an accuracy of $\frac{1}{10}$th of a unit, although E A is adjustable only to units. In like manner, if we make A B 100 times as large as B C, then every unit of resistance in E A represents $\frac{1}{100}$th of a unit in C E; for in this case

$$\text{C E} = \frac{\text{E A}}{100},$$

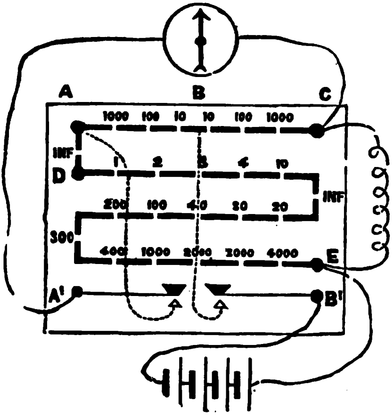
and we can thus determine the value of a resistance to an accuracy of $\frac{1}{100}$th of a unit. In the first case, however, we cannot measure a resistance whose value is greater than $\frac{1}{10}$th of the total resistance we can insert in E A, and in the second case we cannot measure a resistance greater than $\frac{1}{100}$th of the total resistance in E A.

If, now, we wish to measure a resistance which is greater than the total resistance we can insert in E A, we must make B C larger than A B. If B C be made 10 times as great as A B, we can then measure any resistance which is not greater than 10 times the resistance we can insert in E A; but as in this case one unit in E A represents 10 units in C E, we can only be certain of the value of C E within 10 units. In like manner, if we make B C 100 times as great as A B we can measure any resistance which is not greater than 100 times E A, but we can only determine its value within 100 units.

The practical method of joining up the bridge is shown by Fig. 12. When the connections are made, and the proper plugs removed from A B and B C, the *right*-hand key must be pressed down to put on the battery current. Plugs are now removed from E A until we have inserted a resistance, as near as we can guess, equal to the resistance we are going to measure. The left-hand key is next pressed down, and plugs removed or shifted from E A until no movement of the galvanometer needle is produced upon raising and depressing the left-hand key.

If the galvanometer used is a very sensitive one, with a filar suspension, the left-hand key must not, at first, be pressed firmly down, but only snapped down sharply; for otherwise, if equilibrium is not nearly produced when it is depressed, there is a danger of breaking the fibre of the galvanometer

FIG. 12.



needle by the violent deflection. When, however, after repeated trials, we have nearly obtained equilibrium, then the key may be firmly depressed, and the final adjustment of plugs made.

If we know approximately what the resistance we are measuring is likely to turn out, we can calculate, in the way before shown, what would be the best resistances to employ in A B and B C. If, however, we have no idea what the resistance is likely to be, the best thing to do is to take out, say, the 100 plugs, and then, having adjusted E A so as almost to obtain equilibrium, to change them for the 1000 or the 10 plugs, and see if the deflection is thereby increased. The plugs which give the best result being found, we can obtain exact equilibrium by finally adjusting E A, which gives the value of the resistance to be determined. We can adopt the same device when measuring, with unequal proportions, in A B and B C. Thus we can try whether 10 and 100 give a better result than 100 and 1000.

Besides the method of joining up, as shown by Fig. 12, we may also join up by placing the battery between A and C, and the galvanometer between B and E; this renders the action of the galvanometer more sensitive than by the common arrangement, under certain conditions.

Before proceeding to consider the principal uses of the bridge principle, we will explain a method of measuring the resistance of a galvanometer, and also a method of measuring the resistance of a battery, which we have not yet been able to consider.

From the principle of the bridge it will be readily understood that if balance is not established, and a current is flowing through the galvanometer, any change in its resistance will affect the strengths of the currents flowing in all the four branches. If, on the other hand, equilibrium *is* produced, we may take away the galvanometer, thereby disconnecting the two points which it connects, or join those two points together without affecting the currents in the branches. On this principle depends Sir William Thomson's method of measuring the resistance of galvanometers.

The galvanometer is placed in one of the branches of the bridge system, and in the place of the regular galvanometer a key is inserted, which enables us to connect or disconnect the two points where it would have been. The battery, which is in its usual position, causes the galvanometer needle to be deflected.

If now the coils are adjusted, so that this deflection remains the same, whether the key is depressed or not, then we know that equilibrium in the system is produced, and consequently the resistance of the galvanometer is given by the regular proportion

$$\mathrm{EA} = \mathrm{CE}\,\frac{\mathrm{AB}}{\mathrm{BC}}.$$

The connections would be the same as those given by Fig. 12 (p. 66), with the exception that A′ and C, instead of having a galvanometer between them, would be joined by a piece of wire. The galvanometer to be measured would of course be inserted between C and E.

The method of finding the resistance of a battery, known as *Mance's method*, although, perhaps, not strictly coming under the head of a bridge system, is to all appearance so like one that its explanation here may be excused. Referring to Fig. 11 (p. 60), the following is Mance's system:—

The battery, of electromotive force E, whose resistance (C E) is required is placed between the points C and E, and in the place of the battery between B and E, a key is inserted, which, as in the case of the galvanometer test, answers the purpose of connecting or disconnecting the points B and E.

Let us first suppose the key to be up and the points B and E to be disconnected.

The current (C) flowing through the galvanometer will be

$$C = \frac{E}{CE + AE + \dfrac{(BC + AB)G}{(BC + AB) + G}} \times \frac{BC + AB}{(BC + AB) + G}$$

$$= \frac{E(BC + AB)}{G(CE + EA + BC + AB) + (CE + EA)(BC + AB)}.$$

Next let us suppose B and E to be joined by the key being pressed down.

Then the current (C_1) flowing through the galvanometer will be

$$C_1 = \frac{E}{CE + \dfrac{\left(\dfrac{EA \, . \, AB}{EA + AB} + G\right) BC}{\left(\dfrac{EA \, . \, AB}{EA + AB} + G\right) + BC}} \times \frac{BC}{\left(\dfrac{EA \, . \, AB}{EA + AB} + G\right) + BC}$$

$$= \frac{E(AB + EA)BC}{G(BC.AB + CE.AB + CE.EA + EA.BC) + AB.BC.CE + AB.BC.EA + BC.EA.CE + AB.CE.EA}.$$

Now suppose the resistances to be so adjusted that the deflection of the galvanometer remains the same, whether the key is depressed or not; that is, make $C = C_1$, then

$$\frac{E(BC + AB)}{G(CE + EA + BC + AB) + (CE + EA)(BC + AB)} =$$

$$\frac{E(AB + EA)BC}{G(BC.AB + CE.AB + CE.EA + EA.BC) + AB.BC.CE + AB.BC.EA + BC.EA.CE + AB.CE.EA},$$

therefore

$$\frac{BC + AB}{(CE + EA)(BC + AB)} = \frac{(AB + EA)BC}{AB.BC.CE + AB.BC.EA + BC.EA.CE + AB.CE.EA};$$

and by multiplying out and cancelling

$$AB.CE.EA = BC.EA^2;$$

therefore

$$CE = \frac{BC.EA}{AB},$$

which gives the resistance of the battery at once.

The great advantage of this method, which is perhaps the best of any, is that the electromotive force of the battery need only be constant during the very short interval of time occupied in depressing and raising the key.

We will now proceed to consider some of the tests more in direct connection with the bridge system.

When we are measuring the *conductivity* resistance of a wire, whose farther end is not at hand, we should join one end to terminal C, put the farther end to earth, and also put terminal E to earth, and measure in the usual way. It is always as well, however, when possible, to measure without using an earth, by looping two wires together at their farther ends, the nearer ends being joined to terminals C and E respectively; this gives the joint conductivity of the two. Errors consequent from earth currents, or a defective earth, &c., are thereby avoided. We cannot, however, by this means obtain the conductivity resistance of each wire separately. If, however, we have three wires at hand, we can by three measurements obtain the conductivity resistance of each wire, without using an earth. This is effected as follows:—

Let the three wires be numbered respectively 1, 2, and 3. First loop wires 1 and 2, and let their resistance be R_1. Next loop wires 1 and 3, and let their resistance be R_2. Lastly, loop 2 and 3, and let their resistance be R_3. Supposing the respective resistances of 1, 2, and 3 to be r_1, r_2, and r_3, we get

$$r_1 + r_2 = R_1$$
$$r_1 + r_3 = R_2$$
$$r_2 + r_3 = R_3$$

Then subtracting the third equation from the second, we get

$$r_1 - r_2 = R_2 - R_2,$$

and adding this to the first equation

$$2r_1 = R_1 + R_2 - R_3;$$

therefore

$$r_1 = \frac{R_1 + R_2 - R_2}{2};$$

in like manner we should find that

$$r_2 = \frac{R_1 + R_3 - R_2}{2},$$

and

$$r_3 = \frac{R_2 + R_3 - R_1}{2}.$$

For example.

The conductivity resistance of each of three wires, Nos. 1, 2, and 3, was required. Nos. 1 and 2 being looped, the resistance (R_1) was found to be 300 ohms. Nos. 1 and 3 looped gave a resistance (R_2) of 400 ohms. Lastly, Nos. 2 and 3 looped gave a resistance (R_3) of 500 ohms; then

Ohms.

Resistance (r_1) of No. 1 wire $= \frac{300 + 400 - 500}{2} = 100$

Resistance (r_2) of No. 2 wire $= \frac{300 + 500 - 400}{2} = 200$

Resistance (r_3) of No. 3 wire $= \frac{400 + 500 - 300}{2} = 300$

By this device, then, we are enabled to eliminate all sources of error without making a greater number of measurements than would be required if we measured each wire separately, by using an earth.

In taking the *insulation* resistance of a wire, the connections would be the same as for conductivity resistance, only the farther end of the wire instead of being put to earth would be insulated.

It sometimes happens that we want to find the insulation resistance of two sections of one wire, but we can only test from one end.

Suppose A C to be the wire which we want tested for insulation resistance from A in two sections, A B and B C. Let a be the insulation resistance of the section A B, and b the insulation resistance of the section B C; and suppose x to be the insulation resistance of the whole wire from A to C, then we have

$$x = \frac{a\,b}{a + b},$$

from which

$$b = \frac{a\,x}{a - x}.$$

All we have therefore to do, supposing we are testing from A, is first to get the end C insulated, and to measure the insulation resistance, which gives us x. Now get the wire separated at B, and the end of the section A B insulated. Then again measure the insulation resistance, which gives us a, from which b can be calculated.

For example.

The *insulation* resistance (x) of the whole wire, from A to C, was found to be 6000 ohms, and from A to B (a) 24,000 ohms. What was the insulation resistance (b) of the section B?

$$b = \frac{24000 \times 6000}{24000 - 6000} = 3000 \text{ ohms.}$$

To get the *conductivity* resistance of one section of a wire when the resistance of the other section, and also of the whole wire is known, we have only to subtract the resistance of the one section from the resistance of the whole section. The truth of this is obvious.

When the conductivity resistance of a line of telegraph is measured by having the farther end of the line put to earth, the errors arising from the presence of earth currents may be eliminated by making two measurements, one with a zinc and another with a copper current, and taking the mean of the two results. This mean value, although not strictly the correct resistance of the line, is not far wrong when the two results do not differ very largely from one another. To get the exact value from the two measurements is a tedious process, requiring a knowledge of the resistance and electromotive force of the testing battery, and also the electromotive force of the earth current, which must, of course, be constant at the time the measurements are being made.

By earth currents we mean the currents set up by electrical disturbances over the surface of the earth, and also currents due to the polarization of the earth plates.

When the battery connections for measuring conductivity are made, as shown by Fig. 12, in order to put the zinc current to line, we should put the cable to C and the earth to E. To put the copper to line we can either reverse the battery or put the cable to E and the earth to C, whichever is most convenient to the experimenter.

When we are measuring the conductivity of a sub-marine cable, which requires to be carefully done, the best method to adopt is the following:—Put on the battery current for half a minute by pressing down the key; at the expiration of that time, proceed to adjust the plugs, pressing down the left-hand key as required until equilibrium is produced; continue to adjust, if the needle does not remain at zero, and at the expiration of half a minute note the resistance. Now reverse the battery connections, put on the current for half a minute; again measure, again reverse and measure, and so on until about a dozen measurements with either current have been taken. It will usually be found that about half the measurements made with the negative current are the same, and also half the measurements made with the positive current; these results may be taken as the correct measurements, and the mean of the two gives the required resistance.

If the earth at either the testing or the farther station be bad, i. e. if they have a resistance, the error cannot be eliminated by testing with the two currents. It is therefore necessary to know, before making a conductivity test, whether both earths are good, otherwise the measurements are useless.

The presence of earth currents can be detected when the line, galvanometer, and earth are joined to the resistance box, by pressing down the left-hand key alone. This will cause the galvanometer needle to be deflected if there are any currents present. A line is seldom if ever quite neutral in this respect.

It is almost immaterial what battery power is used in measuring conductivity; sufficient, however, should be used to obtain a good deflection on the galvanometer needle when equilibrium is not exactly produced. It is not advisable to use a very low power, for if the earth currents be at all strong the positive and negative measurements may differ largely. About 30 or 40 cells is a convenient number to employ. There is no danger of heating the resistance coils with such a power if the battery be a Daniell charged with plain water, or even a Leclanché, as their internal resistance is considerable. It would not be advisable, however, to use a Grove or a Bunsen battery, or a Daniell charged with acidulated water, as their heating power is great in consequence of their small internal resistance.

CHAPTER VIII.

TESTING FOR FAULTS.

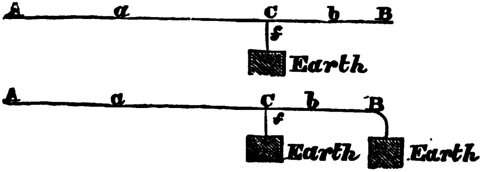
THE theoretical methods of testing for the localities of faults are comparatively simple, but their practical application presents some difficulties.

The simplest kind of fault to localize is a complete fracture where the fault offers no resistance, and the conductivity resistance gives at once its position. Thus, a line which was 100 miles long, and in its complete condition had a resistance of 1350 ohms, that is to say, a resistance of $\frac{1350}{100} =$ 13·5 ohms per mile, gave a resistance of 270 ohms when broken. Then distance of fault from testing station was

$$\frac{270}{135} = 20 \text{ miles.}$$

When the fault has a resistance, the localization becomes more difficult. The following are the theoretical methods generally adopted (Fig. 13).

FIG. 13.



First Method.—Let A B be a line which has a fault f at C, A being the testing station. A first gets B to insulate his end of the line. He then measures the resistance, which we will call l, then

$$a + f = l;$$

therefore

$$f = l - a. \qquad [1]$$

Next, B puts his end to earth, and A again measures. Let the new resistance be l_1, then

$$a + \frac{bf}{b + f} = l_1. \qquad [2]$$

Calling L the resistance of the line, we have also

$$a + b = \mathrm{L};$$

therefore

$$b = \mathrm{L} - a. \qquad [3]$$

From these three equations we have to determine a. Substituting the values of f and b from [1] and [3] in [2], we get

$$a + \frac{(l - a)(\mathrm{L} - a)}{\mathrm{L} + l - 2a} = l_1,$$

therefore

$$a^2 - 2al_1 = \mathrm{L}l - \mathrm{L}l_1 - ll_1;$$

from which

$$a = l_1 - \sqrt{(l - l_1)(\mathrm{L} - l_1)}.$$

For example.

A faulty cable, whose total conductivity resistance when perfect was 450 ohms (L), gave a resistance of

350 ohms (l) when the farther end was insulated, and 270 ohms (l_1) when the end was put to earth. What was the resistance of the conductor up to the fault?

$$\text{Resistance} = 270 - \sqrt{(350-270)(450-270)} = 150 \text{ ohms}.$$

If the length of the cable was 50 miles, then conductivity per mile equals $\frac{450}{50} = 9$ ohms, and distance of fault from testing station consequently equals $\frac{150}{9} = 15\frac{5}{9}$ miles.

Second Method.—Two measurements are made, one by station A, and the other by station B, A and B insulating their end in turn. Thus resistance measured from A when B insulates as before, is

$$a + f = l. \qquad [1]$$

Resistance measured from B when A insulates

$$b + f = l_2, \qquad [2]$$

also

$$a + b = \text{L}, \qquad [3]$$

subtracting [2] from [1]

$$a - b = l - l_2,$$

and adding [3]

$$2a = \text{L} + l - l_2;$$

therefore

$$a = \frac{\text{L} + l - l_2}{2}.$$

For example.

A faulty cable, whose total conductivity resistance when perfect was 450 ohms (L), when measured from A with the end at B insulated, gave a resistance of 350 ohms (l); and when measured from B with the end A insulated, a resistance of 500 ohms (l_2). What was the resistance of the conductor from A to the fault?

$$\text{Resistance} = \frac{450 + 350 - 500}{2} = 150.$$

So far the testing is simple; the practical application, however, presents some difficulty. This is owing to the variation of the resistance of the fault when the testing current is put to the cable in consequence of this current acting on the copper conductor, and through the agency of the sea water, covering it with a salt, which besides increasing the resistance of the fault, also sets up a current opposing the testing current. To make a proper test, then, it is necessary so to manipulate the testing apparatus and battery as to get rid of the polarization and resistance set up by the salt formed on the fault, and to measure the resistance at the moment this is done. The following is known as:—

Lumsden's Method.—The further end of the cable being insulated, the conductor is cleaned at the fault by applying a zinc current from 100 cells for ten or twelve hours, the current being occasionally reversed for a few minutes. A rough resistance test is then made with a copper current.

A positive current is now applied to the cable for about two minutes, using two or three cells for every 100 units of resistance which have been measured. This coats the conductor with chloride of copper.

The cable is now again connected to the resistance box and the connections made as shown by Fig. 12, that is to say, the zinc pole to terminal B′ and the copper to terminal E, the cable to C, and earth to E.

Both keys being depressed, the galvanometer needle is carefully watched and plugs are inserted and shifted unit by unit, so as to keep the needle at zero; for the action of the negative current is to clean off the chloride of copper, and thereby to reduce the resistance of the fault. At a certain point this decomposition becomes complete, and the needle of the galvanometer flies over with a jerk, showing that a disengagement of hydrogen has taken place at the fault which enormously increases its resistance. The resistance in the resistance box at that moment is the required resistance.

The fault being once cleaned by the application of the 100 cells for ten or twelve hours, it is unnecessary on repeating the measurement, which should always be done, to apply the battery for so long a time; ten or twenty minutes, or even less, will generally suffice.

When the measurement is made with the farther end of the cable to earth, the same process of preparation can be employed.

Before making the test, A must of course arrange with B, or *vice versâ*, at what time and for how long he is to insulate, put to earth, &c., his end of the cable.

Practice is required before these tests can be satisfactorily made. An artificial line, however, can easily be made with resistance coils to represent the resistance of the line up to the fault, and a short piece of cable core which has been pierced with a needle for the fault itself. This piece of core is immersed in a vessel of sea water, using a piece of galvanized iron plate or wire for an earth. By this means a very fair idea of the difficulties encountered in testing for faults in cables may be obtained, and good practice made.

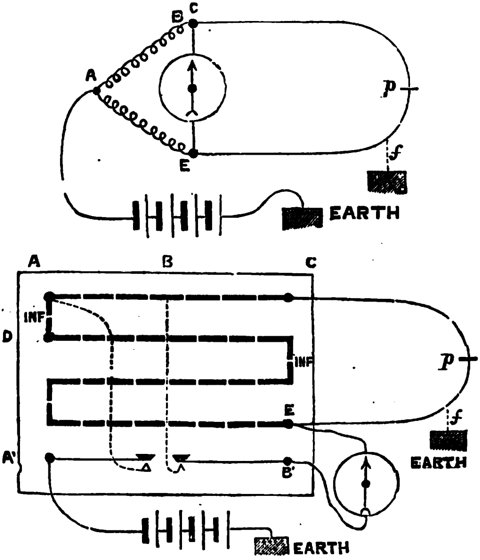
The Loop Test.—When a faulty cable is lying in the tanks at a factory so that both ends of it are at hand, or when a submerged cable can be looped at the end farthest from the testing station with either a second wire, if it contains more than one wire, or with a second cable which may be lying parallel with it, as is often the case, then the simplest and most accurate test for localizing the position of the fault is the loop test.

This test is independent (within certain limits) of the resistance of the fault, thus doing away with the necessity of cleaning and depolarizing which would be necessary in the ordinary tests.

There are two ways of making this test with the form of apparatus hitherto described.

First Method.—Fig. 14 shows the theoretical and practical arrangements. *p* is the point where the

FIG. 14.



two wires or cables are looped together at the farther station, *f* being the fault.

Let x be the distance from C to the fault, y the distance from E to the fault. Then A B and E A being adjusted until equilibrium is produced,

$$AB \times y = EA \times x.$$

Let L be the total conductivity resistance of the whole loop, then

$$x + y = \mathrm{L},$$

therefore

$$x = \mathrm{L} - y.$$

Substituting this value of x in the above equation, we get

$$\mathrm{A\,B} \times y = \mathrm{E\,A}\,(\mathrm{L} - y),$$

from which

$$y = \mathrm{L}\left(\frac{\mathrm{E\,A}}{\mathrm{E\,A} + \mathrm{A\,B}}\right).$$

To obtain L, we should simply join up for the ordinary conductivity test, as shown by Fig. 13 (p. 77). The fault in this case has no effect upon the test, provided it is not caused by the complete fracture of the cable, for in such a case the broken ends become covered with salts, which would make the resistance appear higher than it really is. When, however, the fault is due to a simple imperfection in the insulating sheathing, the ordinary conductivity test gives the correct result.

It is advisable to keep a record of the conductivity resistance, so that it can be ascertained without the necessity of making a measurement.

In the practical execution of this loop test, the connections being made as shown by the figure, all the plugs between B and C must be inserted. This is necessary, because the galvanometer connection is made on to the terminal B′, which is the same as B instead of on to C. The test could be made by placing the galvanometer on to C, but in that case we should lose the advantage of the key, which it is always best to use.

The plugs being inserted between B and C, and the other plugs being in their places, we should remove the 1000 plug from between A and B, and having pressed down the left-hand key, to put the battery current on, which should be a *zinc* one as shown, we should adjust the plugs between D and E, pressing down the right-hand key as required until equilibrium is produced. The different resistances being inserted in the formula, y is found, which being divided by the conductivity resistance per mile of the cable, gives the position of the fault.

For example.

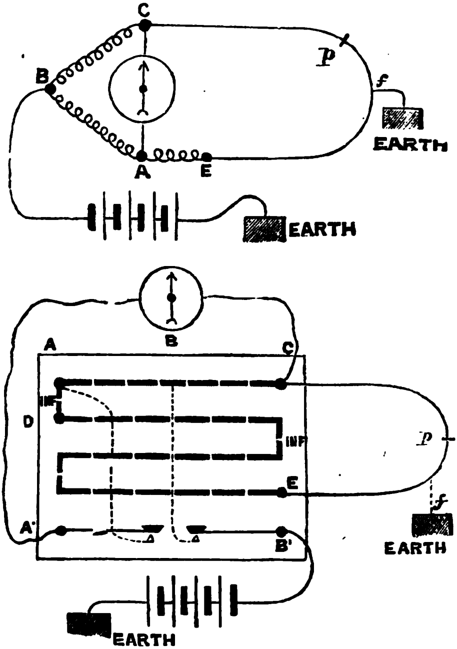
A cable 50 miles long, whose total conductivity resistance was 450 ohms, that is, 9 ohms per mile, was looped with a second cable, whose length and conductivity resistance were the same as the first cable—the resistance of the loop being $450 \times 2 = 900$ ohms. The adjusted resistance in E A to obtain equilibrium was 200 ohms, A B being 1000 ohms; then

$$y = 900\left(\frac{200}{200 + 1000}\right) = 150 \text{ ohms.}$$

Dividing this by the conductivity per mile, which is 9 ohms, we get distance of fault from testing station $= \frac{150}{9} = 15\frac{5}{9}$ miles.

Second Method.—This is shown theoretically and practically by Fig. 15.

FIG. 15.



In this test, B C and A B are usually made equal, E A being adjusted until equilibrium is produced. Then, y and x being the same as before,

$$BC(EA + y) = AB\,x,$$

and

$$x = L - y;$$

therefore

$$BC\,(EA + y) = AB\,(L - y),$$

from which

$$y = \frac{AB \times L - BC \times EA}{AB + BC}.$$

If $AB = BC$, then

$$y = \frac{L - EA}{2}.$$

For example.

The two cables being of the same length and conductivity as in the last example, equilibrium was obtained by making $EA = 600$; then

$$y = \frac{900 - 600}{2} = 150 \text{ ohms.}$$

It is necessary that the faulty one of the two looped cables be attached to E, or else it would be impossible to obtain equilibrium. If we were testing a looped cable, and after having joined it up found that we could not obtain equilibrium, we may be sure that the fault lies between C and p. The cable must then be reversed, and a fresh test made.

The two looped cables may have different conductivities per mile. In this case we must divide the 150 ohms by the conductivity per mile of the cable in which y is measured.

When the position of a small fault—that is to say, a fault which has a high resistance—in a short piece of cable or underground work, has to be localized by the loop test, it is necessary to use a galvanometer which has a low resistance. For if its resistance is high, the short looped wire acts as a short circuit, and very little current can pass through the galvanometer; still less can pass through it if the fault has a high resistance. If, then, it is required to localize faults in short lengths of cable or underground work, such, for instance, as the underground work in the streets of large towns, it is advisable to have a galvanometer specially constructed for the purpose, of thick wire.

Correction for the Loop Test.

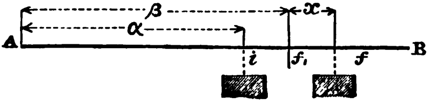
It sometimes happens that the fault in a cable has a resistance approaching that of its insulation resistance when sound; then the position of the fault indicated by the loop test will not be its true position. The reason of this is that the current flowing in a faulty cable has two paths open to it: one through the fault and the other through the whole of the insulating sheathing. The cable, in fact, possesses two faults: the actual fault, and the fault due to the conducting power of the insulating sheathing. This second or *resultant* fault, as it is called, in a homogeneous cable is equivalent to a fault in the centre of the cable whose resistance is equal to the insulation resistance of the cable itself when in good condition. If the cable is not homogeneous throughout, this resultant fault will lie away from the centre.

Its position can be found, however, by the ordinary loop test when the cable is sound.

We have then to determine the true position of the fault when the position and resistance of the resultant fault, the insulation resistance of the cable when imperfect, and the position of the fault indicated by the ordinary loop test, are known.

In Fig. 16, let A B be the cable joined up for the loop test, as shown by Fig. 15, f the actual fault, i the resultant fault, and f_1 the apparent position of the fault given by the loop test.

FIG. 16.



Let P equal the resistance of i, that is, the insulation resistance of the cable when perfect; I the insulation resistance when the cable has a fault, which resistance is due to the joint resistances of the fault, which we will call c, and the insulation P: that is,

$$I = \frac{Pc}{P+c}; \text{ whence } c = \frac{PI}{P-I}.$$

Let $Af_1 = \beta$ and $Ai = a$, therefore $if = \beta - a$.

Now the greater the resistance P becomes with respect to c, the smaller the distance f_1f becomes; and finally when c equals nothing, f_1f equals nothing. Therefore we have, resistance of resultant fault P × distance between f_1 and f = resistance of actual fault c × distance betweeen f_1 and i.

Let distance $f_1\ f = x$, then,

$$P \times x = \frac{P\,I}{P - I}(\beta - a);$$

therefore

$$x = \frac{I}{P - I}(\beta - a),$$

which gives us the position of the true beyond the apparent fault.

Or the distance of the fault from A will be

$$\beta + \frac{I}{P - I}(\beta - a) = \frac{\beta P - a I}{P - I}.$$

For example.

In a looped cable, whose total length was 100 miles, and total conductivity resistance 900 ohms, the ordinary loop test showed the apparent position of a fault which existed in it to be 700 ohms from A, that is,

$$\beta = 700.$$

The position of the resultant fault given by the loop test when the cable was new was found to be 500 ohms from A, that is,

$$a = 500.$$

The insulation resistance of the cable when new was 3,000,000 ohms, and when faulty 600,000 ohms, that is,

$$P = 3{,}000{,}000$$
$$I = 600{,}000,$$

Where was the true position of the fault?
Distance of fault from A

$$= \frac{700 \times 3{,}000{,}000 - 500 \times 600{,}000}{3{,}000{,}000 - 600{,}000} = 750 \text{ ohms.}$$

That is to say, distance of fault beyond distance given by loop test was

$$750 - \beta = 50 \text{ ohms.}$$

Or, supposing the cable to have a resistance of 9 ohms per mile, the true distance of the fault beyond the apparent distance was $\frac{50}{9} = 5\frac{5}{9}$ miles.

If the cable be homogeneous throughout, the resultant fault will appear in the middle of it. In this case a will equal $\frac{L}{2}$, where L is the total length of the loop.

If in the equation,

$$\text{Distance of fault from A} = \frac{\beta P - a I}{P - I}$$

we put $P = I$, or, what is the same thing, $a = \beta$ then,

$$\text{Distance of fault from A} = \beta,$$

as in the ordinary loop test.

In making the first loop test, it is advisable to use as high numbers as possible in A B and E A, because then we can determine the position of the fault to a greater degree of accuracy than when low numbers are used. For, supposing we started with 10 ohms in A B, and obtained equilibrium very nearly with 5 ohms in E A, then the only other resistance we could insert would be 4 or 6 ohms—we could have no finer adjustment; but if we used 1000 ohms in A B, and obtained adjustment nearly with 500 ohms, it is evident that there are 100 numbers from 500 to 400, or from 500 to 600, which we could use to get exact equilibrium.

CHAPTER IX.

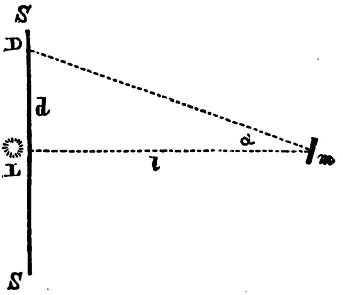
THE THOMSON GALVANOMETER.

THE accuracy with which measurements can be made depends chiefly upon the sensitiveness of the galvanometer employed in making those measurements. The Thomson reflecting galvanometer supplies this requisite sensitiveness, and is the instrument which is almost invariably employed when great accuracy is required, and also when very high resistances have to be measured. Its principle is that of employing a very light and small magnetic needle, delicately suspended within a large coil of wire, and of magnifying its movements by means of a long index hand of light. This index hand is obtained by throwing a beam of light on a small mirror fixed to the suspended magnetic needle, the ray being reflected back on to a graduated scale. This scale being placed about 3 feet distant from the mirror, it is obvious that a very small angular movement of the mirror will cause the spot of light reflected on the scale to move a considerable distance across it.

The needle being very small, and being placed in the centre of a large coil, the tangents of its deflections are approximately directly proportional to the strength of the currents producing them.

In Fig. 17, let L be a lamp which throws a beam upon the mirror *m*, which has turned through a small angle, and reflected the beam on the scale at D. Let *d* be the distance through which the beam has moved on the scale from zero point at L, and let *l* be the distance between the scale and the mirror.

FIG. 17.



Now the angle through which the beam of light turns will be twice the angle through which the mirror turns. This is clear if we suppose the mirror to have turned through 45°, when the reflected beam will be at 90°, or at right angles to the incident beam. If, then, we call a the angle through which the beam of light turns, $\frac{a}{2}$ will be the angle through which the mirror will have turned. Let $\frac{a_1}{2}$ and $\frac{a_2}{2}$ be the two angles through which the mirror has been turned by two currents, of strengths c_1 and c_2 respectively, then

$$c_1 : c_2 :: \tan\frac{a_1}{2} : \tan\frac{a_2}{2},$$

therefore,

$$c_1 : c_2 :: \frac{\sqrt{1+\tan^2 a_1}-1}{\tan a_1} : \frac{\sqrt{1+\tan^2 a_2}-1}{\tan a_2},$$

$\sqrt{1+\tan^2}$ being positive, as the angles are less than 90°.

l being the distance of the scale from the mirror, let d_1 and d_2 be the distances traversed on the scale by the beam of light, then

$$\tan a_1 = \frac{d_1}{l},\ \tan a_2 = \frac{d_2}{l},$$

therefore,

$$c_1 : c_2 :: \frac{\sqrt{1+\frac{d_1^2}{l^2}}-1}{\frac{d_1}{l}} : \frac{\sqrt{1+\frac{d_2^2}{l^2}}-1}{\frac{d_2}{l}};$$

therefore,

$$c_1 : c_2 :: d_2\left(\sqrt{l^2+d_1^2}-l\right) : d_1\left(\sqrt{l^2+d_2^2}-l\right);$$

when d_1 and d_2 do not differ largely, then we may take

$$c_1 : c_2 :: d_1 : d_2;$$

but when this is not so the error may be considerable. For instance, suppose $d_1 = 150$, and $d_2 = 300$. According to the last formula this would show that one current is just twice as strong as the other, but by the correct formula, taking $l = 1500$ divisions (which would be about its value), we find that

$$c_1 : c_2 :: 300(\sqrt{1500^2+150^2}-1500) : 150(\sqrt{1500^2+300^2}-1500),$$

that is

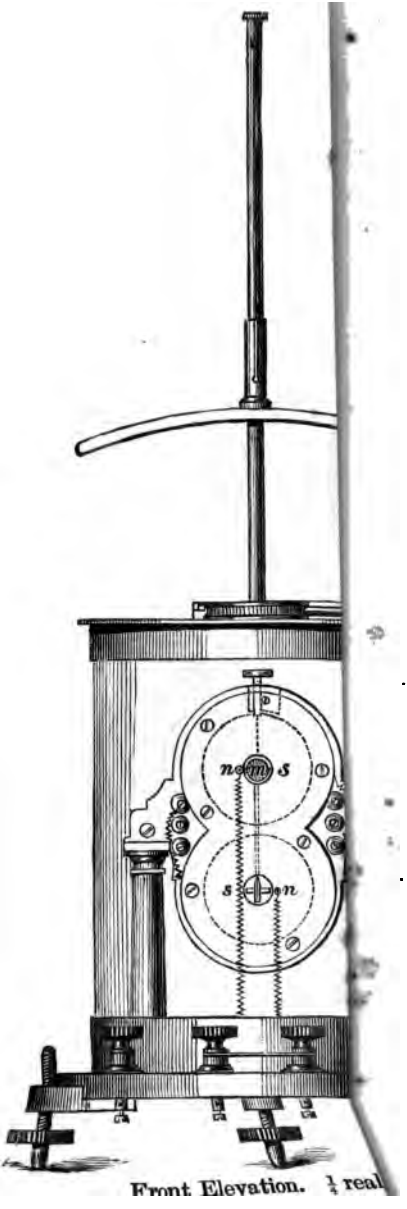
$$c_1 : c_2 :: 2244 : 4456,$$

or

$$c_1 : c_2 :: 150 : 297\cdot 84;$$

so that when extreme accuracy is required we cannot take the strengths of currents as being exactly proportional to the number of divisions of deflection on the scale.

The galvanometer, as usually constructed, consists essentially of a very small magnetic needle, about three-eighths of an inch long, fixed to the back of a small circular mirror, whose diameter is about equal to the length of the magnet. This mirror, which is sometimes a plano-convex lens, of about 6 feet focus, is suspended from its circumference by a single cocoon fibre devoid of torsion, the magnetic needle being at right angles to the fibre. The mirror is placed in the axis of a large coil of wire, which completely surrounds it, so that the needle is always under the influence of the coil at whatever angle it is deflected to. A beam of light from a lamp placed behind a screen, about 3 feet distant from the coil, falls on the mirror, the bottom of which is slightly in advance of the top, and is reflected back on to a graduated scale placed just above the point where the beam emerges from the lamp. The scale is, as



Front Elevation. ¼ real

we have before said, straight, and is usually graduated to 360 divisions on either side of the zero point.

The Thomson galvanometer is made in such a variety of forms that it will be impossible to consider them all; we purpose therefore only to describe one in very common use.

Fig. 18 gives a front and a side elevation (with glass shade, &c., removed) of this form.

It consists of a base formed of a round plate of ebonite, provided with three levelling screws; two spirit-levels, at right angles to one another, are fixed on the top of this plate, so that the whole instrument can be accurately levelled. Sometimes one circular level only is provided, but the double level is much the best arrangement.

From the base rise two brass columns, between which a brass plate is fixed, rounded off at the top and bottom. Against the faces of this plate are fixed the coils (*c*, *c*, *c*, *c*) of the instrument. The brass plate has shallow countersinks on its surface for the faces of the coils to fit into, so that they can be fitted in their correct places without trouble or danger of shifting. Round brass plates press against the outer surfaces of the coils by means of screws, and keep them firmly in their places. There are two round holes in the brass plates coinciding with the centre holes in the coils.

The coils themselves, which are four in number, are wound on bobbins of thin insulating material, the wire being heaped up towards the cheek of the bobbin which bears against the brass plate. This heaping up is done in accordance with a law of Sir William Thomson, so as to obtain, as far as possible, a maximum effect out of a minimum quantity of wire. The edges of the coils are covered with shellac, so as to protect the wire from injury.

Within the holes in the brass plate are placed two little magnets, *n s* and *s n,* formed of watch-spring highly magnetized; they are connected together by a piece of aluminium wire, so as to form an astatic pair of needles. A small groove is cut in the brass plate, between the upper and lower hole, for the aluminium wire to hang freely in.

In front of the top needle is fixed the mirror. It is suspended by a fibre attached, at its upper end, to a small stud, which can be raised or lowered when required. When pressed down as far as it will go the needles rest on the coils, and the tension being taken off the fibre, there is no danger of breaking it by moving the instrument.

One end of each coil is connected to one of the four terminals in front of the base of the instrument, the other ends being connected to one another through the medium of the little terminals placed midway on either side of the coils.

The connections are so made that, when the two middle terminals on the base of the instrument are *joined* together, the whole four coils are in the circuit of the two outer terminals, so that they all four act on the magnetic needles. Some better arrangement for connecting the four coils together than that at present adopted is highly desirable, so that they could be coupled up in series, and the resistance of the galvanometer thereby be reduced to one-fourth the resistance of one of the coils, that is, one-sixteenth of that of all the coils together.

By connecting the first terminal on the base with the third, and the second with the fourth, the coils would be so coupled up as to reduce the resistance of the coils to one-fourth of the total resistance of all the coils together.

Over the coils a glass shade is placed, from the middle of the top of which a brass rod rises. A short piece of brass tube slides over this rod, with a weak steel magnet, slightly curved, fixed at right angles to it. This magnet can be slid up or down the rod, or twisted round, as occasion may require. For fine adjustments a tangent screw is provided, which turns the brass rod round, and with it the magnet.

We have said that the mirror is sometimes made of a plano-convex lens. This is done so as to obtain a sharp image of the spot of light on the scale. The width of the spot can be regulated by means of a brass slider fixed over the hole in the screen, through which the beam emerges from the lamp.

A much better arrangement than the spot of light is now provided with most instruments. The hole through which the light emerges is made round, about the size of a sixpence, with a piece of fine platinum wire stretched vertically across its diameter. A lens is placed a little distance in front of this hole, between the scale and galvanometer, so that a round spot of light, with a thin black line across it, is reflected on the scale. This enables readings to be made with great ease, as the figures on the scale can be very distinctly seen. (The mirror in this arrangement may be a plane one.) When the spot of light only is used, it is necessary to partially illuminate the scale with a second lamp.

To set up the Galvanometer.—It is essential, before proceeding to set up the instrument for use, to see that the ebonite base is thoroughly dry and clean, so that there may be no leakage from the wires to interfere with the tests taken. Indeed, it is as well to place the galvanometer and the other apparatus to be used on a large sheet of gutta-percha or ebonite, more especially if the room in which the tests are to be made is at all damp. Sometimes little ebonite cups are provided for the levelling screws of the instrument to stand in, which answers the purpose of insulating very thoroughly.

The instrument should be set up on a very firm table in a basement story. It is almost useless to *test* with it in an upper room, as the least vibration sends the spot of light dancing and vibrating to and fro. At all cable works the instrument is placed on a solid brick table built on the earth, so that no vibration can possibly affect it.

A suitable table being chosen, place the galvanometer so that the two front levelling screws stand north and south, the front facing west. This is best done by drawing a line on the table exactly north and south, and placing the screws on it.

The levelling screws must next be adjusted until the bubbles of the level or levels show the instrument to be perfectly level.

Now remove the glass shade, and gently raise the stud at the top of the coils by squeezing the tips of the fingers between the head of the stud and the top of the brass plate in which it runs. If the stud is raised by a direct pull there is almost a certainty of its coming up with a jerk and breaking the fibre. On no account must the stud be twisted round, except to get rid of any torsion which may exist in the fibre when it has been replaced after becoming broken.

The stud being raised sufficiently high to allow the mirror to swing clear of the coils, replace the glass shade, screw the brass rod with the magnet, on to its top, and set the magnet about half way up the rod, the poles being placed so as to assist in keeping the magnetic needles north and south.

The scale lamp being lighted, place it in position on the scale stand, the *edge* of the wick being turned towards the brass slider which regulates the width of the beam of light. Having opened the slider to its full extent, the scale and lamp should be placed about 3 feet from the galvanometer, so that it stands parallel with the faces of the coils and that a line drawn at right angles to the scale from the lamphole will pass through the centre of the galvanometer. The reflected beam of light should then fall fairly on the scale. If too high, this may be remedied by propping up the scale, and if too low, by screwing up the levelling screws of the galvanometer. It is easier to prop up the scale than to lower the galvanometer by means of the levelling screws, if the light is too high on the scale.

The spot of light being obtained accurately at the zero point on the scale by turning the regulating magnet by means of the screw, the spot of light should be focussed, by advancing or retreating the lamp and scale, until a sharply defined image is obtained on the scale. The width of the slit may then be diminished, by means of the brass slide, until a thin line of light only is obtained on the scale. If the round spot of light with the line across it is used, the focussing must be made so that the black line of light is sharply defined.

The position of the scale and galvanometer being once obtained, their positions on the table may be marked for future occasions, or, at least, the exact distance of the scale from the galvanometer noted, so that it can be placed right without trouble.

The instrument being now ready for use, if not required to be made sensitive, place the regulating magnet low down; if, on the contrary, it is required to be sensitive, place it high up.

To obtain the *maximum sensitiveness*:—Raise the magnet to the top of the bar, and then turn it half round, so that its poles change places. The magnet will now be opposing the earth's magnetism, and consequently will tend to turn the magnetic needles round. If the magnet is at the top of the rod, the effect of magnetism of the earth on the magnetic needles will be more powerful than the magnetism of the regulating magnet, and the needles will tend to keep north and south; but by placing the regulating magnet lower down a point is reached where the earth's magnetism is just counteracted. Under these conditions the needles will stand indifferently in any position. By placing the regulating magnet about an inch higher than the position which gives this exact counteraction, the magnetism of the earth will be just sufficient to keep the magnets north and south, and consequently the spot of light at the zero on the scale, and at the same time leaves them free to be moved by a very slight force. It will be noticed with the regulating magnet in this position, that in order to get the spot of light at the zero point, the magnet must be turned in the opposite direction to that in which it is required that the needles should move.

It is not advisable to adjust the instrument too sensitively, because it is difficult then to keep the spot exactly at zero, as any slight external action may throw it a degree or two out.

The height of the table upon which the instruments are placed, when working with a Thomson galvanometer, depends a good deal upon the fancy of the experimenter, but as the work is usually done most conveniently standing up, the table should be higher than an ordinary one; from 3 feet 6 inches to 4 feet is a good average height.

The presence of iron near the instrument is not prejudicial to its correct working, so long as it remains stationary. The experimenter should, however, remove any keys or knives he may have about him, as they very much affect the galvanometer if he moves about much. These precautions may seem too minute, but as the very object of the Thomson galvanometer is to enable measurements to be made with accuracy, all likely causes of disturbance should be avoided.

If the fibre of the instrument by any chance gets broken the top front plate must be unscrewed, when the coil which it secures can be removed, and the mirror and its appendages got at. Care should be taken, when replacing the fibre, that only a single thread from the cocoon silk is used, or the sensitiveness of the instrument will be much diminished. The operation requires care, and must be done in a room free from draughts. When the ends of the fibre are passed through their respective holes, and tied, a small drop of shellac varnish may be dropped on them, which will prevent their becoming loose.

It is as well to let the needles remain suspended for a time, so that the fibre may become stretched to its normal length before being used.

The suspending stud should always be pressed down before removing the instrument.

A resistance box, containing three *shunts*, is provided with the galvanometer, of the values $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th of the resistance of its coils, which values, as we have before shown, are necessary in order to enable us to reduce the sensitiveness of the galvanometer to its $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th part respectively.

FIG. 19.

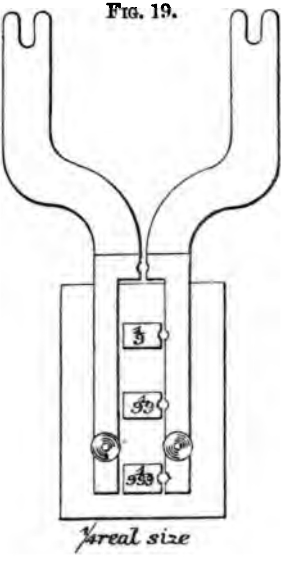


Fig. 19 shows a form of this shunt. By inserting a plug into one or other of the holes, the required shunt is inserted.

The numbers are sometimes marked as $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th, instead of $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th, thereby indicating that the particular shunt reduces the deflections of the needle to that particular fraction, but they have just the same adjustment really in both cases.

The shunts are sometimes enclosed in a round brass, instead of a square wooden box (Fig. 19), which is perhaps a more portable and elegant form.

These shunts, if carefully adjusted, are very useful. Some good electricians, however, object to their use on the ground that their variation in resistance by temperature does not accord with the variation in resistance of the galvanometer, and that consequently they are only correct at the temperature at which they were constructed. Other good electricians, however, use them, and consider them correct.

The two broad strips of copper shown in the figure are used for the purpose of connecting the box with the galvanometer. The blank plug-hole is for the purpose of short-circuiting it, which should always be done when the instrument is not actually in use.

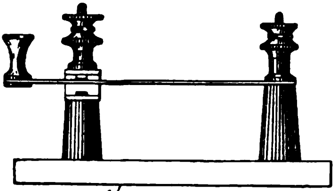
CHAPTER X.

KEYS, SWITCHES, CONDENSERS, AND BATTERIES.

ALTHOUGH the short-circuit plug-hole is convenient to avoid accidental currents being sent through the galvanometer, when the various resistance coils, batteries, &c., are being joined up for making a measurement, yet a key which in its normal condition short-circuits the galvanometer is extremely convenient and useful.

Such a key is represented by Fig. 20. In its normal condition the spring rests against a platinum contact, and, when pressed down, against an ebonite one.

FIG. 20.



¼ real size.

The two terminals of the shunt are connected to the terminals of the key, which in this and most keys are double, so as to enable the wires leading to the resistance coils, batteries, &c., to be also conveniently connected to them.

If it is required to keep the key pressed down for a lengthened period, a small piece of sheet ebonite or guttapercha can be slipped in between the contacts, so as to prevent their making connection when the finger is taken off the key. Some keys of this kind are provided with a catch, which keeps the spring down when it is depressed.

The advantage of the short-circuit key over the short-circuit plug may not seem obvious, but actual practice will soon show the careful experimenter its great use.

Besides the short-circuit key, a *reversing key* is usually inserted in the galvanometer circuit, so that the deflections of the needle may be always obtained on the same side of the scale. A form of reversing key very commonly used is shown in plan and elevation by Figs. 21A and 21B.

FIG. 21A.

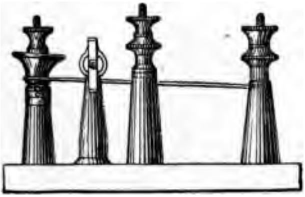
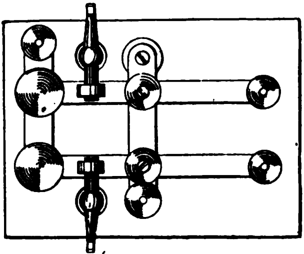


FIG. 21B.



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The galvanometer terminals would be connected to the two end terminals, or, if the short-circuit key is inserted, to the terminals of that key. By pressing down one or other of the springs the current will pass through the galvanometer in one direction or the other. The two handles on either side of the two springs are for the purpose of clamping either of them down when required.

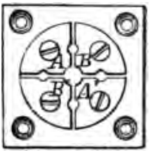
Particular care should be taken, when procuring the keys, to see that the terminals, &c., are not fixed on the top of the ebonite pillars by means of bolts running right through them, as in such a case their advantage is entirely lost, and the terminals might just as well be screwed direct into the base board.

Care should also be taken that the contacts of the keys are clean, as when there are several contacts considerable resistance might be introduced in the circuit from their being dirty.

In addition to the reversing key for the galvanometer, a *reversing switch* for the testing battery is very useful: it need not, however, be such an elaborate one as that used for the galvanometer.

Fig. 22 represents such a switch. It consists of four brass segments screwed firmly down to an ebonite base. Each segment is provided with a screw, to which to attach the testing wires.

FIG. 22.



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In some cases each segment is supported on an ebonite pillar, which improves their insulation very much; and, indeed, would be absolutely necessary for some tests we shall describe.

The poles of the battery would be attached to two opposite terminal screws, say A and A_1, and the leading wires to the two other screws, B and B_1.

To make the current flow in one direction, we should place the plugs between the segments A and B, and A_1 and B_1, and to make it flow in the other direction, between the segments A and B_1, and A_1 and B. If one or both the plugs are removed the battery current will be cut off altogether. It is always best, in order to do this, to remove both the plugs in preference to one only, for if the battery is not well insulated a portion of the current may still be able to flow out of the battery and disturb the accuracy of a test.

Two other pieces of apparatus are necessary to form a very complete set, viz. a "condenser" and a "discharge key."

The *condenser* is merely a Leyden jar exposing a large surface within a small space. It is made of sheets of tin-foil placed in layers between thin sheets of some insulating material, such as guttapercha tissue, paraffined tissue-paper, or paraffined mica. The alternate layers of tin-foil are connected together, so that two sets are formed corresponding to the outside and inside layers of the Leyden jar.

A very convenient form of condenser, manufactured by Messrs. Warden, is shown in plan and elevation by Figs. 23A and 23B.

FIG. 23A.

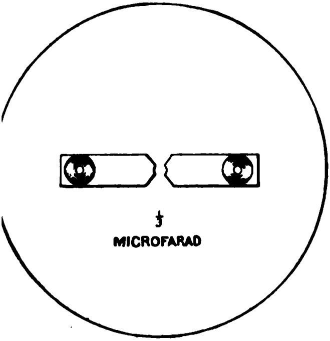
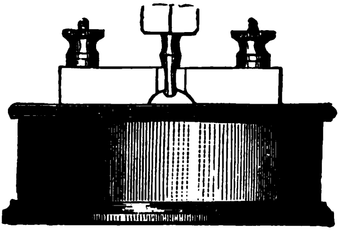


FIG. 23B.



The layers of tin-foil and insulating material (paraffined mica, it is believed) are placed in a round brass box with an ebonite top, on which are fixed the connecting terminals. These terminals are placed on brass blocks, the ends of which are in close proximity to one another, so that a plug can be inserted between them for the purpose of enabling the apparatus to be short-circuited. This should always be done when the condenser is not in use, so that any residual charge which may remain in it after use may be entirely dissipated.

The "electrostatic capacity" of these condensers is usually $\frac{1}{3}$ microfarad, the farad being the unit of electrostatic capacity adopted in this country. They are also made, however, so that several capacities can be obtained, by inserting plugs in different holes. Those having four different capacities, viz. ·1, ·2, ·3 and ·4 microfarads, enable any value from ·1 to 1 to be obtained by inserting one or more plugs. It is very often extremely useful to be able to vary the capacity, so that it is better to have the last form than the first, although it may be a little more expensive.

To enable the discharge from the condenser to be read on the galvanometer a *discharge key* is necessary. This, like the other pieces of apparatus, is made in a variety of forms.

FIG. 24A.

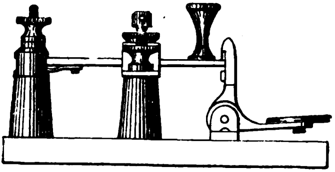
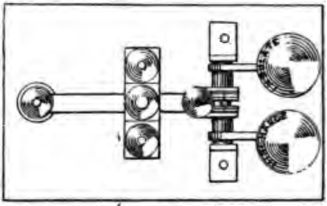


FIG. 24B.



¼ real size

The form shown in plan and elevation by Figs. 24A and 24B is a very convenient and useful one.

It consists of a lever (which should be a solid lever and not a spring), hinged at one end, and playing between two contacts attached to two terminals. Two finger keys, near the other end of the lever, marked "discharge" and "insulate," are connected to two ebonite hooks. The height of the hook attached to the finger key marked "discharge," is a little higher than the other hook, so that the lever stands intermediate between the two contacts when it is hitched against it. When the lever is pressed down against the bottom contact the shorter of the hooks hitches it down. If in this position we press the "insulate" key down, the lever is freed from its hook, and springs up against the second hook, thus insulating the lever from either of the contacts. The "discharge" key now being pressed down, the lever springs up against the top contact.

To the hook of the "discharge" key there is a small piece of metal fixed which is broad enough to come in front of the second hook, so that if the "discharge" key is pressed down first it draws back both the hooks, and thereby, if the lever at starting be hitched to the bottom contact, allows the lever at once to spring up to the top contact. If, however, the "insulate" key be depressed, only the hook attached to that key is drawn back, allowing the lever to spring up against the second hook and be thereby insulated, as at first explained.

Although not perhaps absolutely necessary, it is advisable to have a second set of resistance coils (which need not, however, be of the bridge form) to act as an adjustable shunt for the galvanometer. Also, a simple form of galvanometer to enable the resistance of the Thomson to be quickly taken, is useful.

The form of bridge coils most generally used with the Thomson galvanometer, is that shown by Fig. 4 (p. 15), the keys attached to the other form not being used.

Besides the foregoing, a battery of at least 100 cells is necessary. The form known as the "Sawdust Daniell" is the most convenient, and the one generally used for testing. It consists of an earthenware, or more frequently of a guttapercha jar, about 8 inches high, at the bottom of which is placed a round plate of copper, resting flat. A strip of copper about three-quarters of an inch wide, coated with guttapercha, is fixed to this plate, and brought up the side of the jar. Over this plate a layer of coarsely powdered sulphate of copper is placed; the jar is then filled nearly to the top with damp sawdust, and resting on this is placed a thick disc of zinc, provided with a terminal at the top. A series of these cells is coupled up in the ordinary manner.

The Leclanché battery is used at some cable factories; they have the advantage of great electromotive force, but are not so constant as the *Daniell.*

The batteries should be placed on well-insulated supports, in a dry situation, so as to avoid leakage, which interferes with the constancy of the current.

Besides the large battery, a single cell placed in a small box, with appropriate terminals outside, is required, whose use we shall explain.

CHAPTER XI.

MEASUREMENTS WITH CONDENSERS.

HAVING now described some of the chief instruments used for making delicate measurements and tests, such as would be required for cables, accurate experiments, &c., we shall proceed to show some of their uses and the different ways of *joining up* for various tests.

The instruments being for the purpose of enabling causes of error, which would be inherent in the less delicate forms of apparatus, to be as far as possible eliminated, we must determine the best way of using them.

We have already shown that the deflections on the scale of the Thomson galvanometer, except when they are nearly the same, are not correctly proportional to the current strengths which produce them. To obtain perfectly correct results, then, we must adopt some method of avoiding these widely different deflections. This we can do by using a variable shunt for the galvanometer, and with it obtaining one uniform deflection for all measurements made in one set of tests.

The graduated scale of any galvanometer is not necessarily for the purpose of enabling the strengths of two or more currents to be compared by different deflections, but is also for that of enabling any deflection which may be obtained to be reproduced when required.

It is best to obtain as high a deflection as possible, for then not only will a slight variation from the correct resistance of the shunt cause a greater number of degrees of variation from the deflection required, to be obtained than would be the case if a low deflection was used, but also a higher resistance being required for the shunt, a greater range of adjustment is given to it.

We have previously given several methods of finding the electromotive forces of batteries. Another very simple and quick method is that of charging a condenser from the batteries whose electromotive forces are to be compared, and then noting the discharge deflections. The forces will be directly proportional to these deflections.

Discharge deflections on a galvanometer whose deflections are truly proportional to *constant* currents, unless they are nearly equal, are rarely proportional to the currents which produce them. It is therefore very desirable, in measurements such as these, in order to insure accuracy, to adopt the method we have mentioned, viz. to obtain a uniform deflection by means of a variable shunt to the galvanometer. Thus, if we obtain two similar discharge deflections with two electromotive forces, E_1 and E_2, using shunts of the respective resistances S_1 and S_2; then, since the deflections are the same, the electromotive forces are in the proportion

$$E_1 : E_2 :: \frac{G + S_1}{S_1} : \frac{G + S_2}{S_2},$$

or as the multiplying power of the shunts, G being the resistance of the galvanometer. For if we multiplied the deflection we obtained, by these quantities, we should get the theoretical deflections we should have had if no shunts had been used.

For example.

With an electromotive force E_1 we obtained a discharge deflection of 300° on a galvanometer whose resistance G was 5000 ohms, using a shunt, S_1, of 1000 ohms, and with a second electromotive force, E_2, also a deflection of 300°, using a shunt, S_2, of 2500 ohms; then;

$$E_1 : E_2 :: \frac{5000 + 1000}{1000} : \frac{5000 + 2500}{2500},$$

that is

$$E_1 : E_2 :: 2 : 1.$$

The theoretical deflections we should have obtained had no shunts been used would have been 1800° and 900° respectively, that is

$$\frac{5000 + 1000}{1000} \times 300^\circ, \text{and } \frac{5000 + 2500}{2500} \times 300^\circ.$$

Mr. Latimer Clark, in a communication addressed to the Society of Telegraph Engineers,* points out an error caused by the use of shunts in measuring discharge deflections.

It was found that if a certain discharge deflection was obtained with a shunt, then on removing it the discharge deflection obtained was larger than that given by multiplying the original deflection by $\frac{G+S}{S}$.

After considerable research, the cause of the error was traced to the inductive action of the galvanometer needle on its coils. The movement of this needle set up a slight current, which opposed the discharge current, and consequently reduced its effect. This effect being more marked when the shunt was used, made the discharge deflection without it to appear larger than it should.

The formula for finding what would be the discharge deflection obtained on the removal of the shunt, the discharge deflection without the shunt being given, may be thus arrived at:—

First suppose the shunt to be inserted.

Now in all problems in which a current from a condenser has to be considered, we may suppose the condenser to be a battery with a resistance infinitely great compared with the resistances external to it.

Let E be the electromotive force of the charge,

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* Journal of the Society of Telegraph Engineers, No. 2, vol. ii.

R the resistance of the condenser circuit, G the resistance of the galvanometer, S that of the shunt.

Now let the movement of the needle generate an opposing electromotive force e; then calling C, a and β the respective current strengths in the galvanometer, condenser, and shunt circuits, we get the following equations:

$$a - C - \beta = 0,$$
$$a R + C G - E + e = 0,$$
$$a R + \beta S - E = 0:$$

therefore

$$a R + C G - E + e = 0,$$
$$a R + (a - C) S - E = 0;$$

therefore

$$(R + S)(C G + e) - S(E - C R) = 0.$$

Next suppose the shunt to be removed, and let the intensity in the circuit be C_1, and the new electromotive force generated by the movement of the needle be e_1, then

$$C_1 = \frac{E - e_1}{R + G}; \text{ therefore } E = C_1(R + G) + e_1.$$

Substituting this value of E in the last equation, we get

$$(R + S)(C G + e) - S(C_1(R + G) + e_1 - C R) = 0.$$

Now e and e_1 will be proportional to the deflections of the needle, that is to say, to the strengths of current producing those deflections. They will also be proportional to the strength of the magnetism of the needle, which we will represent by κ.

Then

$$e = \kappa C, \qquad e_1 = \kappa C_1.$$

Substituting these values we get

$$(R + S) C (G + \kappa) - S (C_1 (R + G + \kappa) - C R) = 0.$$

Now R is to be infinite as compared with S and G; therefore dividing both sides by R, and putting R = ∞, we find that

$$C (G + \kappa) - S (C_1 - C) = 0,$$

therefore

$$C_1 = C\left(\frac{G + S + \kappa}{S}\right).$$

To make this formula useful, we must determine the value of κ. This can easily be done thus:—

Provide two condensers, one having exactly twice the capacity of the other. Charge the larger one with a sufficient battery power to obtain a discharge deflection (a_1) of, say, 200 divisions on the scale, with a shunt inserted equal in resistance to the galvanometer.

Now remove the shunt, and having charged the other condenser from the same battery, note the discharge deflection (a_2); let it be 204 divisions.

It will be seen that the deflection we should have obtained with the larger condenser and no shunt would have been $2 a_2$, and this is the theoretical deflection we should obtain when a^1 is multiplied by the multiplying power of the shunt corrected by the constant κ; that is to say,

$$2a_2 = a_1\left(\frac{G + G + \kappa}{G}\right);$$

therefore

$$\kappa = 2\,G\left(\frac{a_2}{a_1} - 1\right).$$

To continue the example we have given, let us suppose G = 5000 ohms; then

$$\kappa = 2 \times 5000\left(\frac{204}{200} - 1\right) = 200.$$

For the particular galvanometer, then, we have been considering, we say that when measuring discharge currents the multiplying power of any shunt (S) which may be used is $\frac{G + S + 200}{S}$.

Suppose we have given, the observed deflection without the shunt and also the observed deflection with the shunt, and we require to know what this latter ought to be in order to give us the *true* deflection compared with the first. Let the true deflection be A; then by the ordinary formula

$$C_1 = A\left(\frac{G + S}{S}\right).$$

But when the error exists

$$C_1 = C\left(\frac{G + S + \kappa}{S}\right).$$

From these two equations we get

$$C = A\left(\frac{G+S}{S}\right)\left(\frac{S}{G+S+\kappa}\right).$$

therefore

$$A = C\left(\frac{G+S+\kappa}{G+S}\right) = C\left(1+\frac{\kappa}{G+S}\right),$$

or in words:

True deflection = observed deflection $\left(1+\frac{\kappa}{G+S}\right)$.

It should be clearly understood that this formula is to be applied to the correction of the deflection obtained *with* the shunt, the deflection *without* the shunt being considered as the index of the current from the condenser.

We may remark that this latter formula corresponds with that obtained by Mr. Charles Hockin, and given by Mr. Latimer Clark in the paper referred to.

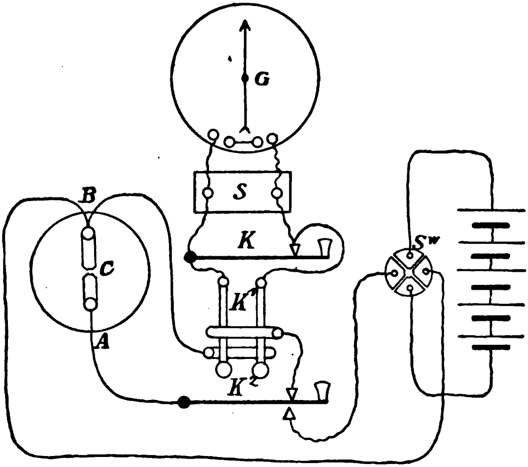
For practical use the formula

$$C_1 = C\left(\frac{G+S+\kappa}{S}\right)$$

is the only one we should require, as by it we can at once determine from the deflection obtained *without* the shunt what the deflection *with* the shunt would be, or *vice versâ*, there being no necessity for making an experimental measurement and then correcting it, which is what is done by Mr. Hockin's formula.

To take these discharge deflections we should make use of the condenser and the discharge key which we have described. The connections would be made as shown by Fig. 25. On pressing down the discharge key K_2, the two poles of the battery are put in connection with the two terminals A and B of the condenser C, and on releasing the key so that it comes in contact with the top contact, the two terminals of the condenser are put in connection with the two terminals of the galvanometer, which thus receives the discharge current through it.

FIG. 25.



If we so arrange the connections that the top contact of the key, instead of being joined to the condenser through the galvanometer, is connected directly to it, and the galvanometer is placed between the back terminal of the key and the second terminal of the condenser; then on pressing down the discharge key we get the current charging the condenser through the galvanometer, which has its needle deflected to one side of the zero point; and then, on releasing the key, we get the discharge deflection, which will be of the same strength as the charge deflection, in the opposite direction to it. The first arrangement, given by the figure, is, however, the one generally employed.

The discharge deflection on the galvanometer is only momentary, the needle or spot of light immediately returning to zero.

In using the Thomson galvanometer for measuring the discharge, the adjusting magnet must be put high up, if it is placed with its poles assisting the earth's magnetism, or low down if it opposes it, so as to make the needle swing slowly enough to enable the deflection to be read on the scale. It is best to avoid making the needle swing very slowly, for then the spot of light will probably not return accurately to zero, but may be three or four degrees out. A little practice will enable a comparatively quick swing to be read to half a degree, or even less.

Besides determining the electromotive force of a battery, we can also determine its internal resistance with great facility by means of a condenser. To do this, first charge the condenser by means of the battery, and note the discharge deflection, which we will call a; next insert a shunt, S, between the poles of the battery; again charge and discharge the condenser, and note the new deflection, which we will call β. Now the amount of deflection produced on the galvanometer needle by the discharge, is the index of the charge in the condenser. Similarly in a condenser, the strength of the charge it takes up indicates the potential of the battery employed to charge it. We may therefore regard the condenser as a galvanometer, but one having a very high resistance. Let us call this resistance R, and let E be the electromotive force of the battery, and r its internal resistance.

First we have, before the shunt is inserted,

$$a = \frac{E}{R + r}, \text{ therefore } E = a\,(R + r);$$

next, inserting the shunt, we have current passing out of battery equals

$$\frac{E}{R + \frac{r\,S}{r + S}},$$

and current passing through galvanometer

$$\beta = \frac{E}{R + \frac{r\,S}{r + S}} \times \frac{S}{r + S},$$

therefore

$$E = \beta \frac{Rr + RS + rS}{S}.$$

Equating the two equations we get

$$\alpha (R + r) = \beta \frac{Rr + RS + rS}{S};$$

therefore

$$\frac{\alpha}{\beta} = \frac{Rr + RS + rS}{RS + rS} = \frac{r + \frac{rS}{R} + S}{\frac{rS}{R} + S},$$

and since G is very great compared with the other resistances, we may put $\frac{rS}{R} = 0$; therefore

$$\frac{\alpha}{\beta} = \frac{r + S}{S},$$

therefore

$$r = S \frac{\alpha - \beta}{\beta},$$

or if S is adjusted till $\frac{\alpha}{2} = \beta$, then

$$r = S;$$

but as it does not follow because β is half α, that therefore the strength of the discharge is halved; or, indeed, that α and β are proportional to the discharges which produce them, we must adopt some other way of making the measurement.

The most accurate method would be the following:—Join up the galvanometer, discharge key, battery, and condenser, as shown by Fig. 24, and shunt the galvanometer until a convenient discharge deflection is obtained on the scale; note this deflection (a) and the resistance of the galvanometer shunt S_1; next insert a shunt S between the poles of the battery of a resistance sufficient to about halve the discharge deflection on the scale. Now increase the resistance of the galvanometer shunt to S_2 until the deflection becomes a, as at first; we then get

$$R = S \frac{a \frac{G + S_1}{S_1} - a \frac{G + S_2}{S_2}}{a \frac{G + S_2}{S_2}} = S \frac{G (S_2 - S_1)}{S_1 (S_2 + G)}.^{*}$$

For example.

With a galvanometer whose resistance (G) was 5000 ohms (which would be about the resistance of a Thomson's galvanometer) we obtained, with the unshunted battery, a discharge deflection of 300, using a galvanometer shunt (S_1) of 1000 ohms. On inserting a shunt (S) between the poles of the battery of a resistance of 240 ohms, which about halved the deflection, it was found necessary to increase the galvanometer shunt to a resistance (S_2) of 2200 ohms, in order to reproduce the original deflection of 300. What was the resistance of the battery?

$$R = 240 \frac{5000 (2203 - 1000)}{1000 (2200 + 5000)} = 200.$$

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[*] In this and the succeeding problems we omit the constant κ for *simplicity.*

An error in this kind of test may possibly arise from one measurement being made with the battery unshunted, when no action goes on in it, and the second being made with it shunted, which may cause a falling off in its electromotive force, as action would be then going on. The accuracy of the test depends upon the electromotive force being constant in both cases.

When comparing large electromotive forces with small ones—as, for instance, 100 cells with 1 cell—the smaller force should be taken first; for a large charge usually leaves a residuum in the condenser, which may be greater than the small force, and which can only be thoroughly dissipated by leaving it (the condenser) short-circuited for some time. If the smaller force is measured first, then any residuum it may leave becomes entirely swamped by the larger force, and no increase of charge is added to the condenser beyond what the force itself possesses.

Although the condenser practically becomes charged instantaneously, it is best to keep the current on for a definite time, such as half a minute or a minute. We shall then be certain that it has become thoroughly charged.

Especial care must be taken to insert a shunt of small resistance in the galvanometer at first, as discharge currents are very liable to demagnetize or weaken the magnetism of the needles when strong currents are used. If this precaution is not taken, a set of measurements for one test may be rendered useless, as a comparison of measurements made before the magnets become weakened with measurements taken after would be obviously impossible, and much loss of time may result.

Electrostatic Capacity.—The measurement of electrostatic capacities by means of a condenser is a very simple operation. To compare condensers of unknown capacity with a standard condenser we have simply to note the discharge deflection (a) given by the standard condenser (F), and then those given by the other condensers (a_1), (a_2), &c.; their capacities (F_1), (F_2), &c., will then be to that of the standard condenser directly as their discharge deflections, taken with the same battery power, or

$$F : F_1 : F_2 :: a : a_1 : a_2.$$

For example.

A standard condenser had a capacity of $\frac{1}{3}$ microfarad, and gave a discharge deflection of 300, and two other condensers, F_1, F_2, gave discharge deflections of 225 and 180 respectively, then

$$\tfrac{1}{3} : F_1 : F_2 :: 300 : 225 : 180;$$

that is,

$$F_1 = \tfrac{1}{3} \cdot \frac{225}{300} = \tfrac{1}{4} \text{ microfarad,}$$

and

$$F_2 = \tfrac{1}{3} \cdot \frac{180}{300} = \tfrac{1}{5} \text{ microfarad;}$$

If we use shunts and obtain the same deflection, then

$$\tfrac{1}{3} : F_1 : F_2 :: \frac{G+S}{S} : \frac{G+S_1}{S_1} : \frac{G+S_2}{S_2}.$$

In measuring the electrostatic capacity of a cable we should have the farther end insulated, the nearer end joined to the back terminal of the discharge key, and the two wires—which in Fig. 24 are shown as joined to one terminal of the condenser—put to earth, the other connections being the same as shown in that figure. Then, as before, the capacity of the cable will be to the capacity of the condenser as the discharge deflection of the one is to the discharge deflection of the other, or obtaining the same deflection by means of the shunts, as the multiplying power of the shunts.

The capacity per mile will be the result divided by the mileage of the cable.

Other methods of measuring the capacity of cables we shall consider hereafter.

CHAPTER XII.

MEASUREMENT OF HIGH RESISTANCES.

THE highest resistance it is possible to measure by means of the Wheatstone bridge we have described is 1,000,000 ohms. It is true that some bridges have another set of resistances in the top row, which will enable the ratio 10 to 10,000 to be used, and consequently a resistance of

$$\frac{10{,}000 \times 10{,}000}{10} = 10{,}000{,}000 \text{ ohms}$$

to be measured, but this is not often the case, and resistances much greater than this frequently require to be measured.

To measure such high resistances a modification of the deflection method we gave in Chapter I. must be adopted.

Provide a single, and also about 100 constant cells. Find their respective electromotive forces by the discharge method given in the last chapter. The electromotive forces of one cell and 100 cells being largely different, a difficulty may arise in doing this.

Suppose we were taking the discharge from the one cell, which, as we have explained, should be taken first, and obtained a discharge deflection of *300, having* adjusted the galvanometer shunt (S_2) for this purpose to 560 ohms. Now, on inserting the 100 cells in the place of the one cell, we should perhaps find that a shunt of 6 ohms gave a discharge deflection of 302, and one of 5 ohms a deflection of 297, or even a greater difference than this. As no nearer adjustment of the shunt is possible, a little extra calculation becomes inevitable. Note the shunt (S_1) which gives the deflection nearest to 300. In this case it will be 6 ohms, which gave 302. Now multiply the 302 by

$$\frac{G + S_1}{S_1},$$

and we get the deflection we should have had, if no shunt had been used. Let this represent the electromotive force of the battery. In like manner, multiply the 300 we obtained with the one cell by

$$\frac{G + S_2}{S_2},$$

and we get the number representing its electromotive force. Giving the numerical values, the ratio of the electromotive force of the 1 cell to the electromotive force of the 100 cells would be

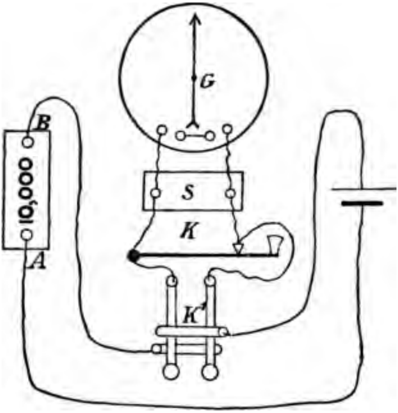
$$\frac{5000 + 560}{560} \times 300 : \frac{5000 + 6}{6} \times 302,$$

or as 2,980 : 252,000.

If now we divide the greater number by the less, we get the value of the 100 cells in terms of the 1 cell. This value we find to be 84·6, that is, the 100 cells is 84·6 times stronger than the 1 cell, and not 100 times. This might arise from some of the cells being defective, or imperfectly insulated. This does not matter, however, as long as we determine, as we have done, how much more powerful the 100 cells is than the one cell.

Having found the value of the 100 cells in terms of the single cell, we next proceed to join up the galvanometer, with a shunt &c., between its terminals,

Fig. 26.



in circuit with a resistance coil and the single cell, as shown by Fig. 26.

Put a resistance of 10,000 ohms in A B (a resistance of 10,000 ohms in a separate box is often used for this measurement), and having first inserted all the plugs in S, press down the short-circuit key, and *proceed to* remove some of the plugs, until a deflection of, say, 300 is obtained, then raise the key and see if the spot of light comes back to zero properly: if it does not, by disconnecting one of the wires, see that the cause is not from the short-circuit key not making proper contact. If this has not the required effect, the adjusting magnet of the galvanometer must be slightly shifted, and, if necessary, put a little lower down, so as to make the needle a little less sensitive. After a few trials this will be satisfactorily done, and the spot of light will always come back to the zero point when no current is passing through the galvanometer.

It may not be possible in this case also to get 300 exactly, but the nearest deflection to it must be obtained and noted.

Suppose it was 301·5, the shunt being 7 ohms. Multiply 301·5 by $\frac{5000 + 7}{7}$, which gives 215,658.

This is the deflection we should get through 10,000 ohms, with no shunt to the galvanometer. There is really in the circuit, besides the 10,000 ohms, the resistance of the one cell, and also the resistance of the galvanometer and shunt combined (which will be of course less than 7 ohms), but this will be so small as to be of no consequence; it may, however, be subtracted from the 10,000 if preferred.

The resistance of the battery can be plugged up in A B before commencing, and also that of the galvanometer and shunt combined when we know what it will be. We should have to proceed as follows:

Plug up the resistance of the battery in the resistance coils A B, and then adjust the galvanometer shunt until a deflection of a little less than 300 is obtained. Calculate what the combined resistance of the galvanometer and shunt will be from the formula $\frac{G\,S}{G + S}$, and plug this value up in the coils, and *then* note what the exact deflection obtained is. If A B is only a single resistance of 10,000, we cannot of course reduce it to allow for these.

Now, if we had used the 100 cells instead of the one cell, our deflection would have been 84·6 times as great as with the one cell. If, then, we multiply 215,658 by 84·6 we shall get the deflection obtainable by the 100 cells through a resistance of 10,000. This value will be found to be 18,244,667. Multiplying this number by 10,000, we get the *constant.*

The process is simplified by using a resistance of 1,000,000 in the place of the 10,000. The constant can then be found with the 100 cells at once.

The object of using the one cell was to enable a readable deflection to be obtained, as 100 cells through 10,000 ohms would throw the spot of light off the scale with the lowest shunt we could use.

Having measured and worked out the constant, which is best done by the help of logarithmic tables, we insert the resistance which is to be measured *in the place* of A B, using the 100 cells in the place of the one cell. Having adjusted S till a deflection of 300, or as near to 300 as possible is obtained, note S and the deflection. Let S be 2500, and the deflection 298. Then the deflection without the shunt would be

$$298 \times \frac{5000 + 2500}{2500} = 894.$$

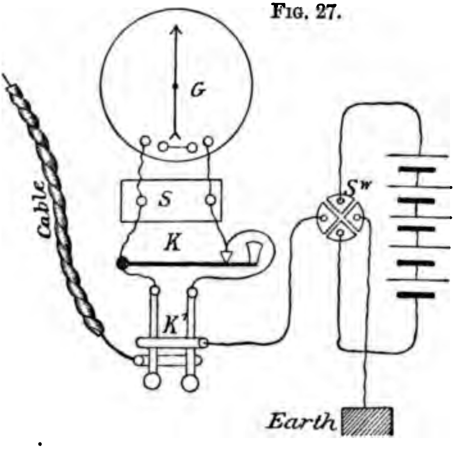
Dividing the constant by this number we get

$$\frac{182{,}446{,}670{,}000}{894} = 204{,}080{,}000 \text{ ohms}$$

as the resistance.

In measuring the insulation resistance of a cable, the constant having been taken, we should join up

FIG. 27.



the galvanometer, shunt, short-circuit key, reversing key, battery switch, battery, and cable, as shown by Fig. 27.

By having both a galvanometer reversing key and a battery switch the trouble of reversing the wires on the galvanometer when the battery current is reversed is avoided, as it can be done more readily by means of the key. The object of reversing the galvanometer connections when the battery is reversed is to obtain the deflection always on the same side of the scale.

The ends of the cable must be trimmed by means of a sharp and clean knife, care being taken that the outer surface of the guttapercha, which has been exposed and oxidized by the air, is completely cut away, and the clean surface thus exposed not touched with the fingers. Some electricians prefer to have the ends then rubbed with a rag soaked in paraffin oil; but this is not absolutely necessary if the trimming is carefully done. Indeed, if the paraffin is not of good quality, it conducts slightly, and may be prejudicial.

The ends being carefully trimmed, and the further end left hanging free, so as not to touch anything, the nearer end of the cable must be attached to the terminal screw of the discharge key, care being taken not to touch the trimmed end in doing so. The switch plugs being inserted, the reversing key, which puts the zinc pole to the cable, must be clamped down, and the short-circuit key being depressed, sufficient resistance inserted in the shunt to obtain a deflection of about 300.

At the end of a minute from the time the key was clamped down, the exact deflection must be noted, and as the spot of light falls down the scale, which it will do owing to what is called *electrification*, the deflections at the end of each minute for about five successive minutes must be noted, the shunt remaining unaltered. The plugs of the battery switch must now be reversed, so as to change the direction of the current, and then the second reversing key must be clamped down, the first one having been released, and deflections again noted at intervals of a minute, as before, until the same number of readings is obtained.

If there is time, it is as well to proceed as follows:

Put the 100 cells direct to the cable for half an hour, reversing the current at intervals of two or three minutes, finishing with the zinc current. Then leave the end to earth for about the same period, and having taken the test with the zinc current, reverse the current for a minute or two, and, finally, having put the cable to earth for about a quarter of an hour, take the test with the copper current.

It will generally be found that the deflections with the copper current will be lower than those with the zinc current, and also that the fall of the deflection in both cases will be quick at first, and will afterwards slowly decrease.

The tests with the cable being finished, the constant may be taken again, and if any difference is observed between it and that obtained at first, the mean of the two should be taken.

The resistance at the end of each minute may now be worked out from the different deflections obtained.

When the cable is connected to the testing instruments by a long leading wire, at the conclusion of the test the end of the lead should be disconnected from the cable, and insulated. If any deflection is observable on the galvanometer when the battery current is put on, this deflection must be subtracted from the deflection obtained when the cable was attached to it.

The ends of the lead must be trimmed in the same manner as the ends of the cable.

CHAPTER XIII.

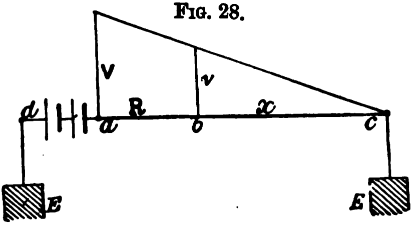
MEASUREMENT OF RESISTANCES BY POTENTIALS.

There are two distinct ways of measuring resistances by potentials:

1st. By noting the potentials at two or more points in a known resistance with which the unknown resistance is in connection, and, by calculation from these, deducing the value required. Or measuring the potential of a battery which is shunted with the unknown resistance, and then reproducing this potential by an adjustable resistance used in the place of the unknown resistance.

2nd. By noting the rate at which a condenser, of a known capacity, discharges itself through the unknown resistance, and calculating the latter from a formula.

Fig. 28.



If we connect a battery to a resistance, R + x, as shown by Fig. 28, the potential of the battery falls regularly along the resistance, being full at *a* and zero at *c*. The same would be the case if *c* and *d* were connected together instead of being put to earth. By similar triangles

$$V : v :: R + x : x,$$

or

$$x = R \frac{v}{V - v}.$$

V being the potential at *a*, and *v* the potential at *b*. So that, if R is a known resistance, we can—by noting the potentials at *a* and *b*—determine the value of *x*.

The potentials are best measured by means of a condenser. To do this we should join up our condenser and galvanometer, as shown by Fig. 25, which we have before given, the only difference being that the terminals which are represented as being in connection with a battery would, instead thereof, be connected to the points *a* and *d* or *c* for determining V, and to *b* and *d* or *c* for determining *v*. The condenser discharges in the two cases give V and *v*.

For example.

If R were 1000 ohms, V 300 and *v* 200, then

$$x = 1000 \frac{200}{300 - 200} = 2000 \text{ ohms}.$$

If *b c* were a portion of a cable making full earth at *c*, we should by this method determine the position of the fault.

Now, in making this test it is not an unimportant question what proportion v should bear to V. If we make it either nearly the value of V, or, on the other hand, very much smaller, we shall find that a slight error (a degree more or less) in reading its value off the scale will make a corresponding error in the value of x very large, when we calculate it from the formula.

It is evident then that there must be some intermediate value of v, such that, if we make any slight error, either of excess or defect in observing its value, the corresponding error in x will be as small as possible.

Now, whatever this value of v may be, R will have a value corresponding to it, so that if we determine what R should be we shall know what v will be.

Let there be an error of excess or defect h in measuring v, and let k equal the corresponding error in x, so that

$$x + k = \frac{\mathrm{R}\,(v + h)}{\mathrm{V} - (v + h)};$$

therefore

$$k = \mathrm{R}\left[\frac{v + h}{\mathrm{V} - (v + h)} - \frac{v}{\mathrm{V} - v}\right] = \frac{\mathrm{R\,V}\,h}{(\mathrm{V} - (v + h)\,(\mathrm{V} -)\,v)};$$

therefore

$$\frac{k}{h} = \frac{\mathrm{R\,V}}{(\mathrm{V} - v)^2}, \text{ since } h \text{ is very small,}$$

$$= \frac{(\mathrm{R} + x)^2}{\mathrm{R\,V}} = \frac{1}{\mathrm{V}}\left[\frac{(\mathrm{R} - x)^2}{\mathrm{R}} + 4\,x\right].$$

When this is a minimum then $\frac{(R - x)^2}{R}$ is a minimum.

But since R is positive, this is so when $R = x$, for then it equals 0.

But when $R = x$ in the formula then $v = \frac{V}{2}$.

So that, if we wish for accuracy, theoretically, the best plan would be to go on adjusting R until $v = \frac{V}{2}$, which makes $x = R$.

There is, however, a practical difficulty in doing this, for we cannot first note V and then adjust R until $v = \frac{V}{2}$, for the alteration in R will have changed V also.

Thus, if, when we commenced, we found $V = 300$, and we then adjusted R until $v = 150$, or half 300, we should probably find that on remeasuring V it would no longer be 300, but some deflection greater or less than 300, according as R was increased or diminished.

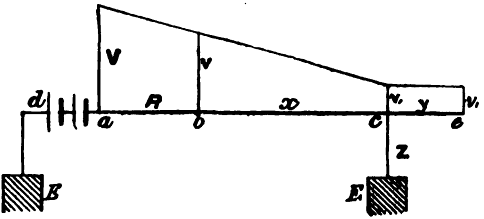
We could, of course, by continual readjustment, at last arrive at a value of R which makes $v = \frac{V}{2}$, but this might involve a considerable expenditure of time; so the best proceeding would be to adjust R by large variations until we get V within, say, 10 of the value of $\frac{V}{2}$, and having noted carefully what the exact values are, to calculate x from the formula.

If, instead of introducing the unknown resistance x, and the known resistance R, between the points *a* and *c*, we join the pole *a* of the battery direct on to *b*, we can determine the value of x by simply noting V, and then inserting an adjustable resistance in the place of x, and altering it until we obtain the potential V at *b*, as at first, when of course $x = R$.

Supposing a cable had a fault which did not make full earth, then the potential would not fall to zero at that point, but would have a value depending upon the resistance of the fault. The potential, however, would be the same as the potential at the farther end of the cable, provided that end were insulated.

If we can determine the value of this potential we can readily detect the position of the fault.

FIG. 29.



Let *b e* be the cable which has a fault at *c*, the end of the cable at *e* being insulated. Then

$$V - v_1 : v - v_1 :: R + x : x;$$

therefore

$$x(V - v_1) = (R + x)(v - v_1);$$

therefore

$$x = R\frac{v - v_1}{V - v}.$$

There would be no difficulty in making this test if the three potentials could be all made in common measure. This could be done if both ends of the cable were at hand; but when a cable is laid it would be impossible, as the potentials at the two ends must be measured with two different galvanometers, and no two galvanometers give exactly the same deflection for the same strength of current, and there is no way of determining the ratio of one deflection to the other with the same strength of current.

The discharge deflections from standard condensers charged from standard electromotive forces taken on the two galvanometers would at once indicate the relative value of their deflections; but unfortunately no satisfactory standard of electromotive force exists, to enable this to be done.

If the cable was sound when the laying was commenced, a battery put on at one end would charge it to the same potential throughout, when the farther end was insulated. Supposing then the charging battery were on the ship, the electricians there would *charge* a condenser from the cable and note the

discharge deflection obtained. A similar observation would be made at the shore end with a condenser of the same capacity, and the deflection obtained on the galvanometer there being noted, the result telegraphed to the ship would put it in possession of the relative value of the deflections of the two galvanometers.

During the laying of the 1866 Atlantic cable these deflections were noted by the shore every five minutes, and the results telegraphed to the ship.

Supposing when the cable was perfect, the shore obtained a discharge deflection of 300, and the ship one of 200, then the ship knows that a deflection of 1 on the shore galvanometer corresponds to a deflection of $\frac{200}{300}$ or $\frac{2}{3}$ obtained on its own galvanometer; all the ship has to do, therefore, to obtain all the results in common measure, is to multiply the results given by the shore by this number.

An example will illustrate how a test would be applied in practice.

Suppose the cable to have a conductivity resistance of 800 ohms.

Let the potential at *a* obtained by the ship be represented by a discharge deflection of 200, and let the result telegraphed by the shore be 300.

Now, suppose a fault to occur, the ship will observe that his discharge deflection has sunk, and he notes its value; let it be 170 (V).

$$V - v_1 : v - v_1 :: R + x : x;$$

therefore

$$x(V - v_1) = (R + x)(v - v_1);$$

therefore

$$x = R\frac{v - v_1}{V - v}.$$

There would be no difficulty in making this test if the three potentials could be all made in common measure. This could be done if both ends of the cable were at hand; but when a cable is laid it would be impossible, as the potentials at the two ends must be measured with two different galvanometers, and no two galvanometers give exactly the same deflection for the same strength of current, and there is no way of determining the ratio of one deflection to the other with the same strength of current.

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CHAPTER XIV.

MEASUREMENT OF RESISTANCES BY FALL OF CHARGE.

THE principle of this method of measurement is that of observing the rate at which a charged condenser of a known electrostatic capacity discharges itself through the unknown resistance, and calculating the latter from a formula which we will now consider.

The elements with which we have to deal are capacity (farad), resistance (ohm), quantity (veber), time (second), and potential (volt).

Let us suppose the condenser has an electrostatic capacity of F farads, and is charged to a potential of V volts so that it contains Q vebers (equal to V F) of electricity, and is discharging itself through a resistance of R ohms during one second.

The quantity of electricity in the condenser at starting is Q vebers.

If now we take a very short interval of time t we may consider the discharge, which really varies continually, to flow throughout that time t, at the same rate as it had at the commencement, and the smaller t is taken, the more accurate will be the result.

Thus since the quantity escaping is directly pro-

$$V - v_1 : v - v_1 :: R + x : x;$$

therefore

$$x(V - v_1) = (R + x)(v - v_1);$$

therefore

$$x = R\frac{v - v_1}{V - v}.$$

There would be no difficulty in making this test if the three potentials could be all made in common measure. This could be done if both ends of the cable were at hand; but when a cable is laid it would be impossible, as the potentials at the two ends must be measured with two different galvanometers, and no two galvanometers give exactly the same deflection for the same strength of current, and there is no way of determining the ratio of one deflection to the other with the same strength of current.

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An example will illustrate how a test would be applied in practice.

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The discharge deflections from standard condensers charged from standard electromotive forces taken on the two galvanometers would at once indicate the relative value of their deflections; but unfortunately no satisfactory standard of electromotive force exists, to enable this to be done.

If the cable was sound when the laying was commenced, a battery put on at one end would charge it to the same potential throughout, when the farther end was insulated. Supposing then the charging battery were on the ship, the electricians there would *charge* a condenser from the cable and note the

discharge deflection obtained. A similar observation would be made at the shore end with a condenser of the same capacity, and the deflection obtained on the galvanometer there being noted, the result telegraphed to the ship would put it in possession of the relative value of the deflections of the two galvanometers.

During the laying of the 1866 Atlantic cable these deflections were noted by the shore every five minutes, and the results telegraphed to the ship.

Supposing when the cable was perfect, the shore obtained a discharge deflection of 300, and the ship one of 200, then the ship knows that a deflection of 1 on the shore galvanometer corresponds to a deflection of $\frac{200}{300}$ or $\frac{2}{3}$ obtained on its own galvanometer; all the ship has to do, therefore, to obtain all the results in common measure, is to multiply the results given by the shore by this number.

An example will illustrate how a test would be applied in practice.

Suppose the cable to have a conductivity resistance of 800 ohms.

Let the potential at *a* obtained by the ship be represented by a discharge deflection of 200, and let the result telegraphed by the shore be 300.

Now, suppose a fault to occur, the ship will observe that his discharge deflection has sunk, and he notes its value; let it be 170 (V).

therefore

$$v = \mathrm{V}\,\frac{100 - n}{100}.$$

Substituting this value of v in the above equation, we get

$$t_2 = \frac{\cdot 30103}{\log \dfrac{100}{100 - n}} \cdot t_1 = \frac{\cdot 30103}{2 \cdot 000 - \log (100 - n)} \cdot t_1 .$$

For example.

If a cable lost 20 per cent. of its charge in 5 minutes; in how many minutes would it fall to half charge?

$$t_2 = \frac{\cdot 30103}{2 \cdot 000 - \log (100 - 20)} \times 5 = 15' \; 32''.$$

From the equations

$$\mathrm{V}\left(\frac{v}{\mathrm{V}}\right)^{t_1} = v_1,$$

$$\mathrm{V}\left(\frac{v}{\mathrm{V}}\right)^{t_2} = v_2$$

we can find what would be the potential, v_2, after a certain interval of time, t_2 having given the potential at first, and the potential v_1, after a time, t_1.

We have from these two equations

$$\left(\frac{v_1}{\mathrm{V}}\right)^{t_2} = \left(\frac{v_2}{\mathrm{V}}\right)^{t_1};$$

therefore

$$v_2 = \mathrm{V}\left(\frac{v}{\mathrm{V}}\right)^{\frac{t_2}{t_1}}.$$

This formula we should have to work out by the aid of logarithmic tables.

And so the formulæ might be expanded and modified to almost any extent to suit various cases, upon which however it is unnecessary to dwell here.

CHAPTER XV.

JOINT TESTING.

JOINTS are the weak points in a cable, and it is therefore essential that they should be not only carefully made but carefully tested.

A joint being a very short length of the core, offers, or should offer, a very high resistance; it would consequently be impossible to test it by a direct deflection method, that is, a method similar to that by which the insulation resistance of a cable is taken. Even with a very powerful battery, the galvanometer deflection, provided the joint be good, would be almost inappreciable.

There are two methods which can be adopted.

A condenser can be charged through the medium of the joint and after a noted time the discharge taken, which gives the amount which has leaked through the joint. This is known as *Clark's accumulation method.*

Or a charged condenser may be allowed to discharge itself through the joint, and the amount lost after a certain time noted.

In both these methods the discharge deflections are compared with the results obtained with a few feet of perfect core.

Accumulation Method.—A guttapercha or ebonite trough is provided, which is suspended by long rods of the same material from any convenient hook.

The good insulation of the trough is of great importance, and consequently the suspending rods should be quite dry and clean. It is usual to give them a rub with a piece of paraffined rag, which prevents the formation of moisture on their surface.

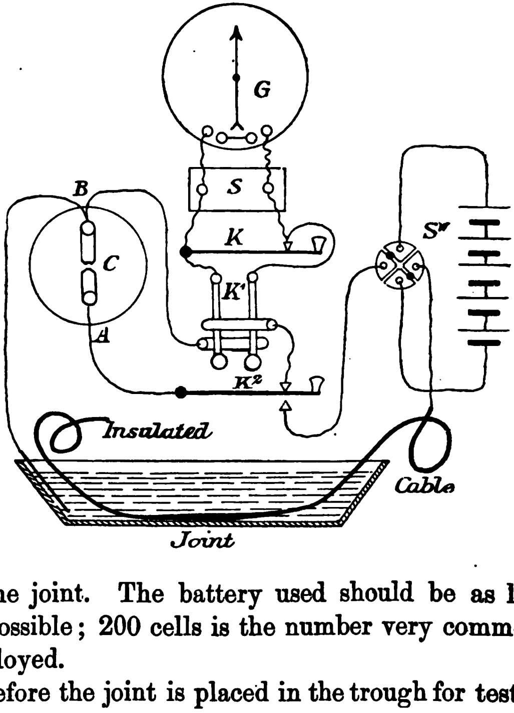
We may here remark that *surface* leakage is almost the only thing to be feared in electrical apparatus, and this should always be seen to by keeping all surfaces over which leakage is likely to occur clean and bright. The peculiar formation of ebonite causes minute quantities of sulphuric acid to form on its surface, which, however, may be removed by washing with clean water. It is as well to do this before rubbing with the paraffined cloth.

The trough is filled with water in which the joint to be tested being immersed is held down by two hooks placed at the bottom.

The portion of the core on either side of the joint should be carefully dried (not paraffined), for the same reason that the suspending rods were so treated.

The connections for the test, shown by Fig. 30, are very similar to those shown by Fig. 25; the only difference being that the pole of the battery, which in that figure was connected directly to the condenser, in the joint test is connected to it through the medium

FIG. 30.



of the joint. The battery used should be as large as possible; 200 cells is the number very commonly employed.

Before the joint is placed in the trough for testing, it is necessary to see that the trough is sufficiently well insulated. This can be done by connecting the *end of* the battery wire, which for the regular test would be attached to the core, to the plate in the trough and then pressing down the discharge key which charges the condenser. The battery wire being then disconnected from the plate, the discharge is noted, after an interval of time equal to that which would be occupied by the test of the joint. This should be equal, or very nearly so, to the instantaneous discharge.

The insulation of the trough being satisfactorily obtained, the joint is immersed in it, and the connections being made, the discharge key is pressed down, when the battery charges the condenser through the joint.

After a certain time, usually one minute, the discharge deflection is noted.

A similar measurement is also made, using about 6 feet of perfect core in the place of the joint.

If, in this case, the discharge deflection after the same interval of time is not less than that obtained from the joint, the joint is considered defective and is re-made.

A thoroughly perfect joint it is evident should test at least as well as an equal length of perfect core, and the fact that so long a length as 6 feet of core is taken as a comparison, shows that the present system of jointing is far from perfect.

When only one observation each is taken with the joint and the core, the results are only roughly comparative, and do not represent accurately the comparative resistances of the two, because if both the discharge deflections were taken after certain equal intervals of time, it would be found that the ratio of the two results would not agree with the ratio that would be obtained if the results were taken after longer or shorter equal intervals of time. By taking, however, a second reading with each, after longer or shorter intervals, true comparative results may be arrived at.

Thus the resistances of the two can be worked out from the formula

$$R = \frac{T}{F \log_e \frac{V}{v}},$$

which formula, we must point out, is as true for the case in which a condenser is *charged* through a resistance as for the case in which it *dis*charges itself through a resistance, the T in the former case being the difference between the times after which the observations were taken. Thus, if the first observation were made after 20 secs., and the second after 30 secs., T would be 30 - 20 = 10 secs.

If we simply require the comparative resistances, then we have

$$R_1 = \frac{T}{F \log_e \frac{V_1}{v_1}},$$

$$R_2 = \frac{T}{R \log_e \frac{V_2}{v_2}}.$$

therefore

$$R_1 : R_2 :: \log_e \frac{V_2}{v_2} : \log_e \frac{V_1}{v_1},$$

V and v being the potentials observed at the beginning and end of the difference of time T, with say the joints, and V_2 and v_2 the potentials observed also at the beginning and end of the difference of time T, with the length of core.

Discharge Method.—This would be precisely similar, in principle, to the method of measuring high resistances by fall of charge given in the last chapter, and the connections would be the same, the one end of the core taking the place of one end of the resistance, and the plate in the trough the place of the other.

The resistance of the joint can then be easily determined.

The system of charging the condenser through the joint cannot of course be carried out unless one end of the core is at hand to which to attach one pole of the battery.

When a joint is made in a cable core at sea neither end can be got at. The joint, however, could be tested by making the connections as for the last method of testing only instead of joining the core to the condenser terminals, putting that terminal to earth and, before the joint is made, arranging with the shore that at a certain time the end of the cable shall be put to earth. The rate at which the charged condenser would discharge itself through the joint could then be observed, and the resistance calculated from the regular formula.

The first method could also be adopted by using an earth in the same way.

We may if we please, in both these tests, place the galvanometer between the back terminal of the key and the condenser, and join the two terminals from which it was removed by a piece of wire. We should then get a charge as well as a discharge deflection, and there is this advantage, that if the joint is very bad or the trough not well insulated, we should get a permanent deflection after the charge deflection has taken place.

The connections should always be so made that the zinc pole is connected to the core and the copper pole to the plate.

CHAPTER XVI.

SPECIFIC MEASUREMENTS.

In order to compare the relative conducting power, insulation resistance, and inductive capacity of the materials used in the construction of the core of submarine cables, it is necessary that they should each of them be referred to some standard unit with which the comparison can be made. For the conductivity of a wire, the relative conductivity of the pure metal may be taken as the standard.

The *specific conductivity* measurement is then a very simple matter.

We have first to measure the resistance of a certain length of pure copper wire of a known weight, and then taking the resistance of our sample copper wire whose specific resistance we require, note its length and weight; we then have all the data we require. For we have to calculate from these, by simple proportion, what would be the resistance of a piece of the sample, of equal length and weight with the pure sample, and then the resistance of the wire experimented on compared with the resistance of the pure sample, which is taken as 100, would be its specific resistance.

For example.

Suppose our pure sample was 30 feet long, and weighed 300 grains, and had a resistance of ·656 ohms, and the length of our impure sample was 20 feet, its weight 500 grains, and its resistance ·200 ohms.

Then since, in the pure sample, 30 feet weighing 300 grains has a resistance of ·656, 1 foot weighing 10 grains will have a resistance of $\frac{\cdot 656}{30}$, and 1 foot weighing 1 grain a resistance of $\frac{\cdot 656 \times 10}{30} = \cdot 2186$.

For the impure sample we have, resistance of 1 foot weighing 1 grain is $= \frac{\cdot 200 \times \frac{500}{20}}{20} = \cdot 2500$.

Then to get the specific conductivity (x) we have the proportion,

$$\cdot 2500 : \cdot 2186 :: 100 : x;$$

therefore

$$x = \frac{\cdot 2186 \times 100}{\cdot 2500} = 87 \cdot 44.$$

To obtain the *specific insulation* resistance of any material is not such an easy matter, for we have no pure standard material with which to compare it, and even if we had the resistance would be so enormously high that we could not, as in the case of the wire, get a piece of a certain length and compare it by measurement with another. We must therefore look for some other method.

Now, the form in which guttapercha is used for submarine cables is that of a cylinder, in which the conducting wire is concentrically placed; and to compare the relative resistances of different cores we must first ascertain the law of the insulation resistance of cores whose sheathings have various thicknesses. As this is an interesting problem, we will give it at length.

Looking at a transverse section, let us suppose the sheathing to be divided into a number of concentric circles, such that the resistance of the piece between any two circles equals ρ. For this to be the case, it is evident that the circles nearer the circumference must be of a greater thickness than those near the centre, since their circumferences are greater.

Let there be n of these circles, so that $n\rho = W$ (ρ here corresponds to the little interval of time t in the fall of charge problem, p. 149).

Now if $\left.\begin{matrix} r_d \\ r_{d+1} \end{matrix}\right\}$ be internal and external radii or diameters of any one cylinder, and if the difference $r_{d+1} - r_d$ is very small, the resistance of the cylinder will be

$$\frac{(r_{d+1} - r_d s)}{2 \pi l r_d}$$

when l is the length of the cable, and s the specific resistance of the insulating material.

Now the smaller we make $r_{d+1} - r_d$, the nearer will this be true. But in order to do this, we must make ρ small and n large. Now

$$\frac{(r_{d+1} - r_d)\,s}{2\,\pi\,l\,r_d} = \rho,$$

since ρ equals the resistance of each cylinder; therefore

$$r_{d+1} = r_d\left(1 + \frac{2\,\pi\,l\,\rho}{s}\right).$$

Then, as in the problem we have referred to,

$$r_n = r_d\left(1 + \frac{2\,\pi\,l\,\rho}{s}\right)^n$$

where r_n or R is the external, and r_d or r the internal radius of the sheathing; that is,

$$R = r\left(1 + \frac{2\,\pi\,l\,\rho}{s}\right)^n = r\left(1 + \frac{2\,\pi\,l\,W}{s\,n}\right)^n,$$

and the larger n is the nearer is this true; therefore make $\rho = 0$, and $n = \infty$ so that $n\,\rho$ still equals W; we then get a perfectly accurate result. Let

$$\frac{2\,\pi\,l\,c\,W}{s\,n} = \frac{1}{x},$$

so that $x = \infty$ when $n = \infty$. Then

$$R = r\left[\left(1 + \frac{1}{x}\right)^x\right]^{\frac{2\pi l}{s}W}$$

when $x = \infty$, but when this is the case the expression within the square brackets is known to be equal to e, thus

$$\frac{R}{r} = e^{\frac{2\pi l}{s} W};$$

therefore

$$W = \frac{s \log_e \frac{R}{r}}{2\pi l} = \frac{s \log \frac{R}{r}}{2\cdot728\, l};$$

therefore

$$s = W \frac{2\cdot728\, l}{\log \frac{R}{r}}.$$

Now if we take for our standard a cable, the core of which has a length of 1 knot, insulation resistance of 1 megohm, and of such external and internal radii or diameters (that is, the diameters of the sheathing and conductor), that $\log \frac{R}{r} = 2\cdot728$, then $s = 1$.

If, then, we measure the insulation resistance per knot, and also the external and internal diameters of our sample core, and insert the values in the formula, we get its specific insulation resistance.

Since we take the data of 1 knot to insert in the formula, we may write it

$$s = W \frac{2\cdot728}{\log \frac{R}{r}}.$$

For example.

The core of a cable had an insulation resistance of 300 megohms per knot. Its external diameter was $\frac{4}{10}$th inch, and internal diameter $\frac{1}{10}$th inch. What was its specific insulation resistance,

$$S = 300 \frac{2 \cdot 728}{\log \frac{4}{1}} = 1360.$$

From what we said in Chapter XIV., it will be seen that the formula for giving the specific inductive capacity (K) of a cable core will be

$$K = F \frac{\log \frac{R}{r}}{2 \cdot 728},$$

where F is the capacity per knot of the core in microfarads. An example is unnecessary.

CHAPTER XVII.

CORRECTIONS FOR TEMPERATURE.

In order to make tests for conductivity, insulation resistance, or inductive capacity strictly comparative, it is either necessary that they be made at the same temperature, or, when this cannot be done, the temperatures at which they are taken should be noted, and a correction made.

Various formulæ have been suggested to enable this to be done, but the following seems to be the most satisfactory one.

It is found, when the temperatures are not very widely different, that for every degree of increase in temperature, an equal percentage of increase in resistance takes place; that is to say, if it increased at a certain rate per cent. by a rise of one degree of temperature, it will be increased by the next degree of rise at the same rate per cent. calculated on the new resistance.

This being so, it will be evident, on consideration, that the percentage of increase for a certain number of degrees will be the same at whatever part of the scale they are taken. Thus, if a resistance increased 25 per cent. between 30° and

40°, it would increase 25 per cent. between 65° and 75°.

If then we take a wire of any metal, and determine how much its resistance is increased by any number of degrees of temperature, we can determine how much the resistance of any other wire of the same metal and quality would be increased.

The law we have stated is exactly the same as the law for the fall of potential in an insulated cable. We have simply, in fact, to substitute resistances for potentials and degrees of temperature for intervals of time in any of the formulæ we had for the above case, and we get our formulæ for change of resistance by change of temperature.

At the end of Chapter XIV. we gave a formula

$$v_2 = V\left(\frac{V}{v}\right)^{\frac{t_2}{t_1}}.$$

If, then, we suppose a resistance to have increased from r to R, by an increase of temperature of n°, and to R_1 by an increase of n_1°, by substitution in the above formula, we get the equation,

$$R_1 = r\left(\frac{R}{r}\right)^{\frac{n_1}{n}}$$

as representing the connection between these quantities.

If we determine R and r experimentally, we can find what R_1 will be for an increase in the temperature of 1°, 2°, 3°, &c.; and by embodying the results in a table, we can determine from an experimental measurement made at any temperature, what the resistance of a wire, of the same kind as that from which the table was constructed, would be at any temperature we may require.

An example will render this clearer:

Suppose we had a wire of copper whose resistance r at 10° was found to be 1000 ohms, and whose resistance R at 60° was 1200 ohms. Then this wire had, by an increase of $60° - 10° = 50°$ (n), increased its resistance by 200 ohms, that is, $\frac{1200}{1000}$, or 1·2 times.

We therefore know that any other wire of similar quality will increase its resistance by that amount, by an increase of 50° of temperature.

This result is noted in our table as the coefficient of *increase* for 50°, by which we must multiply our observed resistance to obtain its value at the required temperature.

The result we have obtained will enable us to determine the increase of resistance for any other number of degrees of temperature, and also the coefficients; for our formula becomes

$$R_1 = 1000 \left(\frac{1200}{1000}\right)^{\frac{n_1}{50^\circ}}.$$

Thus, if we want to find the coefficient for 30° of increase, we have

$$R_1 = 1000\left(\frac{1200}{1000}\right)^{\frac{30^\circ}{50^\circ}} = 1116,$$

showing that an increase of 30° has increased r by 116 ohms, or $\frac{1116}{1000} = 1 \cdot 116$ times, which gives the coefficient of increase for 30°.

The coefficient then being $\frac{R_1}{r}$, we may say

$$\text{Coeff.} = \left(\frac{R}{r}\right)^{\frac{n_1}{n}}.$$

To obtain the coefficient for a decrease in resistance, that is, when we want to find what the resistance will be at a lower temperature than that at which it is measured, we must take the reciprocal of the coefficient for the *increase* of resistance. For it is clear that since to correct the resistance to the higher temperature we *multiply* by the coefficient, to bring it back to the lower temperature, we must *divide* by it, which is the same as multiplying by the reciprocal.

To get the number of degrees of increase or decrease of temperature we have, of course, only to subtract the observed from the required temperature, or *vice versâ*.

The range of temperatures required in practice is not large.

If we calculate for a difference of from 0° to 30° it will usually be sufficient.

It is usual in practice to reduce all measurements to 75° Fahr.

The same law as that we have given answers for the increase in resistance of guttapercha or india-rubber cores.

It will, however, be found that the influence of temperature on the insulating material is very much greater than on metal.

The exact effect of temperature on inductive capacity has not, it is believed, been yet determined or published.

By a reverse process we can tell what the temperature of a wire is if we know what its resistance at one temperature is, and also its resistance at the unknown temperature. For all we have to do is to divide one resistance by the other, and note with what number of degrees of temperature the coefficient so obtained corresponds, and then this result shows the number of degrees the wire has above or below the temperature at which the wire was measured.

For example.

To take the first case, we found that the wire had a resistance of 1000 ohms at 10°, and at the temperature, which we will suppose to be unknown, a resistance of 1200, then the coefficient is $\frac{1200}{1000} = 1 \cdot 2$, which corresponds to an increase of 50°; the temperature of the wire is therefore $50° + 10° = 60°$.

In this way, if we took the resistance of a cable at two different temperatures before it was laid, and constructed a table therefrom, and then after it was laid measured its resistance, we could tell the mean temperature of the sea.

CHAPTER XVIII.

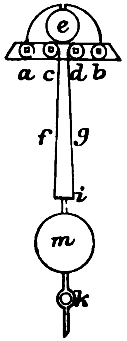
THE THOMSON QUADRANT ELECTROMETER.

THE use of this beautiful instrument is becoming so universal, both in the physical laboratory and in the telegraph cable factory, that no treatise on electrical testing could have any pretension to completeness unless it gave some details of its use and value.

Fig. 31 (p. 180) gives a general view of the instrument.

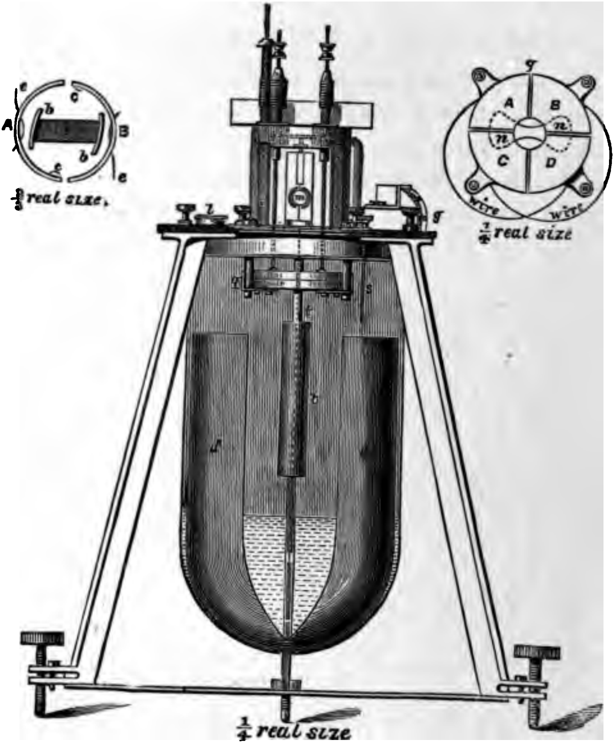
In the small figure to the right, *n n* is a thin needle of sheet aluminium, shaped something like a double canoe-paddle. It is rigidly fixed at its centre to an axis of stiff platinum wire *k* (Fig. 32), in a plane perpendicular to it. At the top end of the wire a small cross-piece *i* is fixed, to the extremities of which single cocoon fibres are attached. These fibres, at their upper ends, are fixed to small screws *c* and *d*, by the turning of which their length can be altered. The small screws *a* and *b* enable the screws *c* and *d* to be shifted either to the right or left. Finally, by turning *e*, the screws *a* and *b* can be

FIG. 32.



parted more or less, thereby separating the threads of suspension, and rendering the tendency of the

FIG. 31.



needle to lie in its normal position more or less powerful.

A little below the cross-piece *i* is fixed the mirror *m*, which reflects its movements on a scale in a manner similar to that of the galvanometer we have before described. The platinum wire below the mirror passes through a small guard-tube *t* (Fig. 31), which prevents any great lateral deviation of the needle and its appendages, which might cause damage should the instrument receive any rough usage.

It will be seen in the figure that the needle is suspended, apparently beneath four quadrants (*q*), A, B, C, and D. There are, however, four quadrants also below the needle, united to the top ones at their circumferences. The arrangement is in fact a round, flat, shallow box, cut into four segments.

The alternate segments are connected together by wires shown in the figure.

Now, if the needle is electrified and the quadrants are in their normal unelectrified condition, and are placed symmetrically with reference to it, no effect will be produced on the needle. That is to say, the spot of light on the scale will be stationary exactly at the centre line.

But if the quadrant D, and consequently A, be electrified, then an attraction or repulsion will be exerted on the needle, causing it to turn through an angle proportional to the potential of the electricity.

As the angular movements are very small, the number of divisions of deflection on the scale will represent the degree of potential which the quadrants have.

We can also connect another electrified body to C and B; the needle will then move under the influence of both forces.

To render the instrument of real practical use, several conditions must be assured.

Let us suppose the needle to be electrified.

We stated that at starting the ray of light should point to the centre line on the scale. To insure this the quadrants must be symmetrically placed. This can be roughly done by hand, as means are provided for enabling the quadrants to slide backwards or forwards, and to be fixed by means of small screws shown in the large figure. For making the final adjustment, one of the quadrants (B) is provided with a micrometer screw (*g*), which enables a very slight motion to be given to it.

We must also have means of keeping the needle at one uniform potential for a considerable time.

The needle itself could only contain a very slight amount of electricity, and a very slight escape of this would seriously lower its potential, and make comparative measurement useless; for it is evident that the whole principle of the instrument depends upon the potential of the needle remaining constant during the time a set of experiments are being made.

To get over this difficulty a large glass jar, like an inverted shade, is provided, which is partially coated with strips of tin-foil (*f*) outside. Inside the jar, to about a third of its height, strong sulphuric acid is placed. This answers a threefold purpose. It enables the air inside it to be kept quite dry, thereby very perfectly keeping those parts insulated which require to be so. Secondly, it holds a charge of electricity (acting as the inner coating of the jar); and thirdly, it allows the charge to be communicated to the needle without impeding its movements. This latter is affected by means of a fine platinum wire, which is attached to the lower end of the thick wire which supports the needle and mirror.

The fine wire dips into the acid, whose charge is thereby communicated to the needle.

To keep this wire from curling up out of the acid, and also to steady the movements of the needle, a small plummet of lead is attached to the end of the wire. This will be seen in the figure.

So far, the jar answers the purpose of keeping the needle supplied with electricity; but although this may prevent the potential from falling very rapidly, it will not prevent its doing so entirely.

As the instrument is extremely sensitive to very slight differences of potential, some means are requisite by which any small loss can be easily supplied without there being any danger of putting in too much.

This is effected by means of the *replenisher*, whose principle we can explain by the help of the small cut to the left in Fig. 31.

A and B are two curved metal shields, one of which (say A) is connected to the acid in the jar and the other, B, to the framework of the instrument, and through it to the foil outside the jar.

b and *b* are two metal wings insulated from one another by a small bar of ebonite, which is centred at *s*, so that it turns in a plane represented by the paper. The spindle is represented in the large figure by *s*, other parts being omitted for simplicity.

It will be observed that the wings curve outwards. This is done in order that they may make a short contact in their revolution with springs *c c* and *e e*. *c* and *c* are connected together permanently, but insulated from the rest of the apparatus. *e* and *e* are connected to their respective shields A and B.

Now let us suppose the wings to be rotated by their ebonite cross-piece in the reverse direction of the hands of a watch.

As soon as the left-hand wing comes in contact with the spring *c* at the lower part of the figure, the right-hand one comes in contact with the other. The two wings being thus connected together, and under the influence of the shields, the electricity in A, which we will call positive, draws negative to the wing close to it, and drives the positive to the other wing.

On rotating a little further the wings clear the springs, and being thus disconnected, each retains its charge.

Continuing their rotation, the right-hand wing, which had the positive charge communicated to it, comes in contact with the spring of shield A, and the charge is communicated to the jar, the negative in like manner on the other wing running to the outer coating of the jar. The shields are now in a neutral condition, as at first, and on continuing the rotation the process is repeated.

Thus every turn increases the potential of the charge in the jar, and by continuing the rotation we can augment this as much as we please.

By reversing the motion we can diminish the charge, if we require to do so.

The axis of the replenisher projects above the main cover, and is easily turned by the finger.

But we still require some arrangement by which we can see whether we have kept the potential constant. This is done by means of a small gauge.

It consists of two metallic discs having their planes parallel and close to each other. The lower of these planes, which will be seen dotted at the upper part of the figure, is in electrical connection with the acid of the jar from which it takes its potential. The upper disc is perforated with a square hole immediately over the centre of the lower disc.

A light piece of aluminium, shaped like a spade, has the part corresponding to the blade fitting in this square hole. At the point where the handle would be joined to the blade this spade is hinged, by having a tense platinum wire fixed to it, which runs at right angles on each side of the handle and blade, and lying in the same plane as the blade.

When the lower plate is electrified, it would attract the blade, thereby raising the end of the handle. So that if we notice the position of the end of the handle with respect to a mark, and see that it moves above or below it, we know that the electricity of the lower plate is either overcoming the tendency of the light platinum wire to keep it up, or is unable to do so.

If then we charge our jar to such a potential that the position of the handle is just by the mark, and we keep it so, we know that the jar is at the same potential. When we notice the handle sinking below the mark, we know that the potential of the jar is sinking; but a few turns of the replenisher will bring it up again.

In the actual arrangement the rung of the handle would represent a fine black hair.

Inside the handle there rises a small pillar, with two black dots on it. The sign of division ÷ represents this, the line being the hair which by the movement of the spade blade rises above or below the two dots, which of course would be almost quite close together.

To enable the hair and spots to be seen distinctly, a plano-convex lens is placed a little distance off. Care must be taken, in order to avoid parallax error, to keep the line of sight a normal to the centre of the lens.

We spoke of the lower disc, which becomes electrified by the jar, and which acts on the spade blade. Now it is evident that if the distance between the plates be always the same, and the elasticity of the platinum axial wire be also the same, to get the hair between the two spots is to obtain the jar at the same potential.

But we may require to get this potential, although the same, whilst a certain set of the experiments are being made, yet different for different series of experiments. This is provided for, by enabling the lower disc to be lowered by screwing it round.

A flat brass plate covers the mouth of the jar, and is secured to it so as to be air-tight and prevent the entrance of moisture.

A kind of lantern rises from the middle, which covers the mirror and its suspending arrangements, and above this a sort of square box protects the gauge.

The front of the lantern is of glass, which allows the ray of light to fall on the mirror and be reflected back on the scale.

Terminal wires are in connection with each set of quadrants. These pass through ebonite columns to the outside of the case, and have appropriate terminals attached to them. They can be pulled up and disconnected from the quadrants if necessary.

A third terminal wire also is brought outside, which can be turned round on its axis. It has at its lower end a small spring, fixed at right angles to it. By turning this terminal wire round, the spring can be brought in contact with the platinum wire of the needle, and consequently with the acid, which can thus be charged. When this is done the spring is moved away, so that no accidental leakage can take place through it.

Various insulating supports are provided inside the jar and lantern. One supports the guard tubes and the adjusting screws of the needle; others support the quadrants.

The whole arrangement is supported by a kind of tripod on a metal base, which keeps it steady There are also levelling screws, and a level *l* on the brass cover, to enable the instrument to be properly levelled, so that the axis of the needle may swing clear of the guard tube.

From the description the necessary adjustments will be fairly well understood.

The length of the suspending fibres must be such that the needle hangs midway between the top and bottom plates of the quadrants. They should be as nearly as possible of equal tension.

Care should be taken that the brasswork inside is free from dust and shreds of cotton, and the glass supports should be made as clean as possible.

It is as well to allow the apparatus to rest for some little time after it has been charged with the acid, to allow the films of moisture to become removed. The acid should be strong.

The jar, under ordinary conditions, will retain its charge so well that it will lose but ½ per cent. per day.

When adjusting the instrument so as to get the light in the centre of the scale, it is as well to connect the terminals of the quadrants together, so as to insure that they may have the same potential should they happen to have retained any charge.

Should the adjustment not be perfect, the micrometer screw must be turned till exact mechanical "zero" is obtained. This zero is not the zero marked on the scale; for, unlike the galvanometer, its scale is graduated from 0 to 720, 360 being the ordinary zero.

The black line across an illuminated disc is used as a rule in preference to the simple spot of light.

To enable high potentials to be measured with the electrometer, an "*induction plate*" is added in the most recent instruments. It consists of a thin brass plate, smaller in area than the top of the quadrant beneath it, and supported from the main cover by a glass stem. It is provided with a terminal similar to the others (not given in the figure). The use of it will be explained.

CHAPTER XIX.

THE USE OF THE ELECTROMETER.

THE instrument being placed on a sheet of ebonite or block of paraffin wax to insulate it, and being levelled up, and the jar being charged by a spark or two from a small electrophorous and the zero obtained correctly, it is ready for use.

There are several ways of making the connections to the terminals of the quadrants, frame, and induction plate, so as to get various degrees of sensitiveness for measuring potentials of various strengths.

The following is the most sensitive arrangement, such as would be used for taking the potential of a Daniell cell :—

One pole of the battery would be connected, through the medium of a reversing key, to one quadrant terminal, and the other to the frame of the instrument and to the second quadrant terminal. This, by reversing the key, would give about 50 divisions on either side of the 360, equal to 100 in all.

Leaving one pole of the battery to the frame, the next degree of sensitiveness is obtained by disconnecting the set of quadrants that are connected to the *frame*, the other connections being the same as in the last. By this arrangement the needle is acted upon by one set of quadrants only.

By using the induction plate we may still further diminish the sensitiveness of the instrument.

When we connect the pole of the battery to a set of quadrants, those quadrants take the potential that it has. But if we connect it to the induction plate, then the charge in the quadrant below is only an induced one, and, since there is an interval between the plate and the quadrant, this induced charge will be small, and the effect on the needle proportionally little.

Again, we may disconnect one set of quadrants, and connect the wire from the battery to the quadrants and its induction plate, which will, as it were, partially *bind* the charge. The effect on the needle will then be less still.

Once more we may connect the battery wire to the induction plate, leaving its quadrants disconnected, and join the other quadrants to the base of the instrument.

Lastly, to get the least sensitive arrangement, we must connect the pole of the battery to the induction plate, the quadrants being both disconnected from their terminal wires.

In all these arrangements the second pole of the battery is kept connected to the frame.

We can, if we please, in each of these cases interchange the terminals of the quadrants, that is to say, use the left terminal where we used the right, and *vice versâ.*

There is one more point to mention in connection with the instrument, and that is, that it may be found, on raising one of the terminals to disconnect it from the quadrants, that the act of doing so causes the image on the scale to deviate a few degrees from zero in consequence of a current being induced thereby.

In the most recent form of instrument there is a small milled vulcanite head provided, by turning which the quadrants are connected to the frame, and the charge being thereby dissipated, the image returns to zero. When this is done the milled head must be turned back again before commencing to test again.

The electrometer can be used in every test where a condenser is usually employed.

In using the condenser we have to charge it, and then note its discharge on the galvanometer, which gives the potential. With the electrometer we have simply to connect to its terminals, the wires which would be connected to the condenser, and the permanent deflection on the scale gives us the potential, which can be observed at leisure.

Thus, in taking the resistance of a battery by the method given on p. 126, we should first connect the battery wires to the electrometer (through the medium of the reversing key is best), note the deflection, then insert the shunt, again note the deflection, and calculate from the formula.

The great value of the electrometer, however, lies in the fact of its enabling us to notice the continuous fall of charge in a cable, and not, like the condenser method, merely determine what the potential has fallen to after a certain time. We can see with unfailing accuracy when the charge has fallen to one-half or any other proportion we please.

We see, in fact, exactly what is going on inside the cable at any moment.

The connections for such a test could not well be simpler. We charge the cable, connect it to the electrometer, the frame being to earth, and then notice the deflection as it gradually falls down the scale. We do not even require a battery, as we can charge the cable with a few sparks from an electrophorous.

The degree of sensitiveness necessary for any particular cable, of course we can only tell by experience.

Joints are readily tested with the instrument. It is unnecessary to trace out the connections in this and other tests, as in all cases we have only to substitute the electrometer in the place of the condenser, and abolish the galvanometer.

The condenser is sometimes not removed, as it increases the capacity of the quadrants as the jar does the needle. It is, however, stated that tests so made are not reliable.

Before concluding we will call attention to a method suggested by Professor Fleeming Jenkin for measuring very high resistances by means of the electrometer.

When very high resistances, such, for instance, as short lengths of highly insulated cable, are measured by the fall of charge method, the fall, even in a considerable time, would be very small, and even if it were somewhat rapid, the accuracy of the test would not be satisfactory, as the difference between the deflection at the beginning of the test, and that after the interval of time must be only a small fraction of the whole length of the scale. And if they are not accurately noted, still less can we be satisfied of the accuracy of our result when worked out from our formula.

Professor Jenkin's improvement consists in virtually prolonging the scale and counting the divisions from an *inferred zero.*

An explanation of his method of making the test will best show what this is.

One set of quadrants is connected to one pole of the battery, and the other pole to the frame of the instrument and to earth.

The second set of quadrants is connected to the cable.

By joining for an instant the two quadrants together, the cable and both sets of quadrants take the same potential, therefore on disconnecting them the needle will be at zero.

The potential, however, of the cable, and the quadrants connected to it will fall, and the needle be deflected.

Suppose, now, one cell connected to the electrometer gave 100 divisions deflection, and suppose the battery which charged the cable was 100 cells, then if the cable lost 1 per cent. of its charge, the charge remaining would be 99, and as the other quadrant, being permanently connected to the 100 cells, has the potential of 100, the difference between the two is $100 - 99 = 1$ cell, which, as we have said, gives 100 divisions. The 2 per cent. loss would give 200 divisions, and so on, whereas by the ordinary method, if we get 300 say at first, 1 per cent. loss would only move the image down to 297, and 2 per cent. down to 294.

When all the charge is lost the deflection would evidently be $100 \times 100 = 10{,}000$, which is the inferred zero. To obtain this zero for any particular battery, we should have to get the deflection from 1 cell and then determine by the method we have before given what the electromotive force of the testing battery is in terms of the 1 cell. Then by multiplying the 1 cell deflection by this we get what we require.

To get the numbers representing the potentials, we must evidently subtract the deflections on the scale from the inferred zero.

To get the full range of the scale we should, at starting, get the image on the actual zero marked on the scale, which is, as we have before said, at the end, and not the middle.

There are many other uses to which the electrometer may be put, which we have not space to describe here, but those who get accustomed to its manipulation will find them out, and will not regret having made themselves masters of the instrument.

TABLE I.

Natural Tangents.

Degrees.	Tangents.	Degrees.	Tangents.	Degrees.	Tangents.
1	·017	31	·601	61	1·804
2	·035	32	·625	62	1·881
3	·052	33	·649	63	1·963
4	·070	34	·674	64	2·052
5	·087	35	·700	65	2·145
6	·105	36	·726	66	2·246
7	·123	37	·753	67	2·356
8	·140	38	·781	68	2·475
9	·158	39	·810	69	2·605
10	·176	40	·839	70	2·747
11	·194	41	·869	71	2·904
12	·212	42	·900	72	3·078
13	·231	43	·932	73	3·271
14	·249	44	·965	74	3·487
15	·268	45	1·000	75	3·732
16	·287	46	1·036	76	4·011
17	·306	47	1·072	77	4·331
18	·325	48	1·111	78	4·705
19	·344	49	1·150	79	5·145
20	·364	50	1·192	80	5·671
21	·384	51	1·235	81	6·314
22	·404	52	1·280	82	7·115
23	·424	53	1·327	83	8·144
24	·445	54	1·376	84	9·514
25	·466	55	1·428	85	11·430
26	·488	56	1·483	86	14·300
27	·509	57	1·540	87	19·081
28	·532	58	1·600	88	28·636
29	·554	59	1·660	89	57·289
30	·577	60	1·732	90	∞

TABLE II.

NATURAL SINES.

Degrees.	Sines.	Degrees.	Sines.	Degrees.	Sines.
1	·017	31	·515	61	·874
2	·035	32	·530	62	·883
3	·052	33	·544	63	·891
4	·070	34	·559	64	·899
5	·087	35	·573	65	·906
6	·104	36	·588	66	·913
7	·122	37	·002	67	·920
8	·139	38	·015	68	·927
9	·156	39	·629	69	·933
10	·173	40	·643	70	·939
11	·191	41	·656	71	·945
12	·208	42	·669	72	·951
13	·225	43	·682	73	·956
14	·242	44	·694	74	·961
15	·259	45	·707	75	·966
16	·275	46	·719	76	·970
17	·292	47	·731	77	·974
18	·309	48	·743	78	·978
19	·325	49	·755	79	·981
20	·342	50	·766	80	·985
21	·358	51	·777	81	·987
22	·374	52	·788	82	·990
23	·391	53	·798	83	·992
24	·407	54	·809	84	·994
25	·422	55	·819	85	·996
26	·438	56	·829	86	·997
27	·454	57	·838	87	·998
28	·469	58	·848	88	·999
29	·485	59	·857	89	·999
30	·500	60	·866	90	1·000

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